

Outline

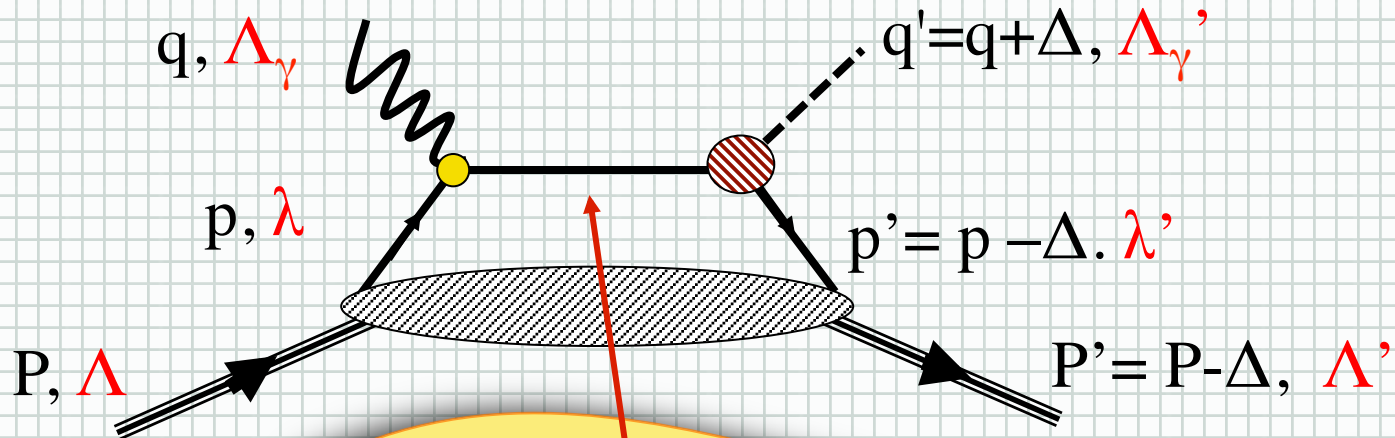
- ✓ Motivation:
 - ✓ How do we detect quarks and gluons Orbital Angular Momentum (OAM)...
 - ✓ Is it an observable...
 - ✓ What are the observables...
- ✓ Introducing a procedure
 - ✓ Transverse Momentum Distributions and Generalized Parton Distributions: unraveling new multiparton correlations
 - ✓ Overview of the type of information one can extract
 - ✓ How reliably can GPDs be measured?
 - ✓ Towards a global fit: models, parameters, theoretical errors, resolution (GGL, PRD 2011)
Can we understand flavor decomposition of Dirac and Pauli form factors?
- ✓ Conclusions and Outlook

How do we detect quarks and gluons OAM?
(Is it an observable?)

A key observation for the detection of angular momentum

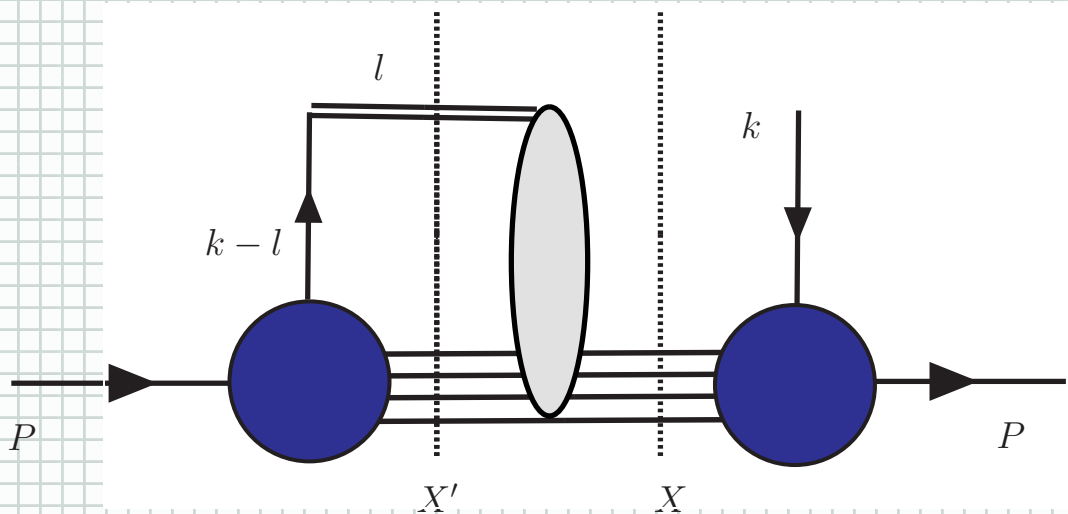
$$\gamma^* p \rightarrow \gamma(M) p'$$

GPDs



Loop directly in LO amplitude allows us to detect Single Spin Asymmetry (SSA)

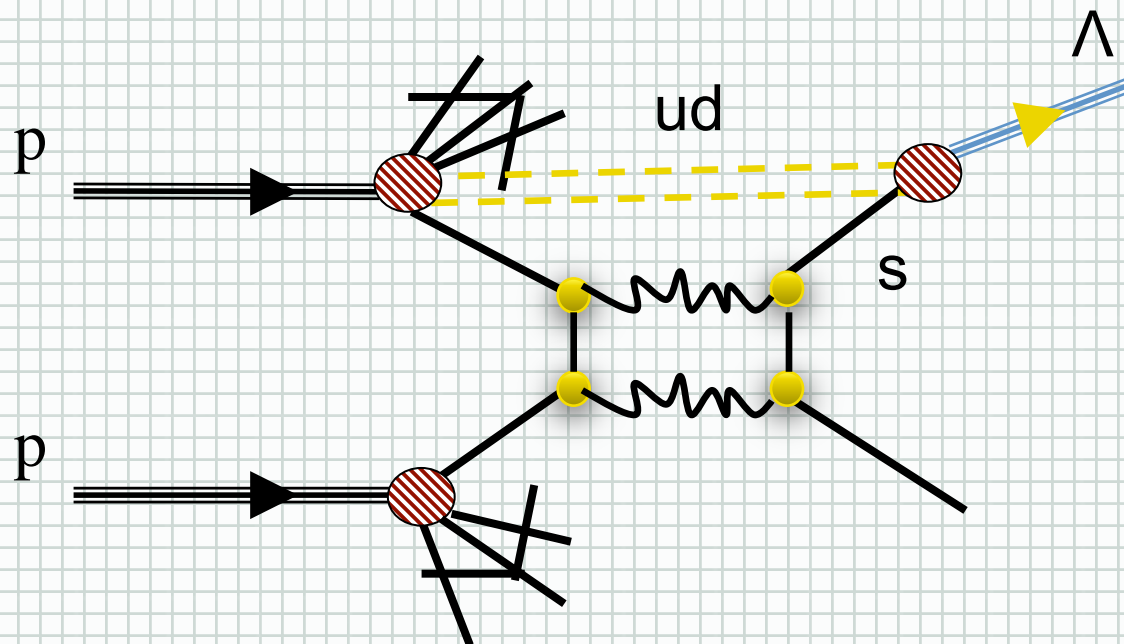
TMDs



$$h_1^\perp, f_{1T}^\perp$$

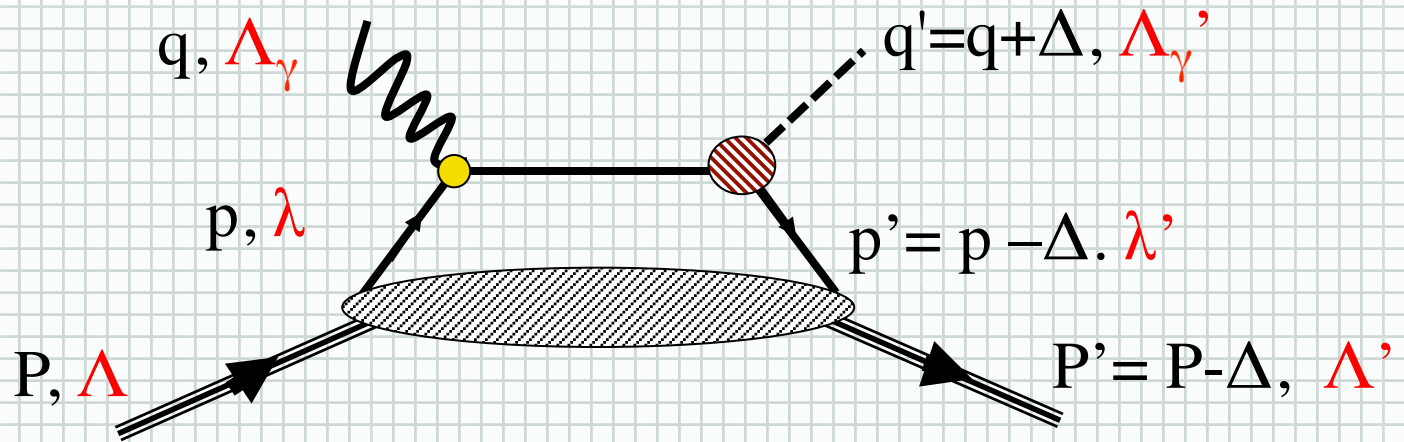
Brodsky, Hwang Schmidt, Ji and Yuan,...

Transverse Spin Asymmetries for Λ in pp scattering



Following Dharmaratna & Goldstein , 90's

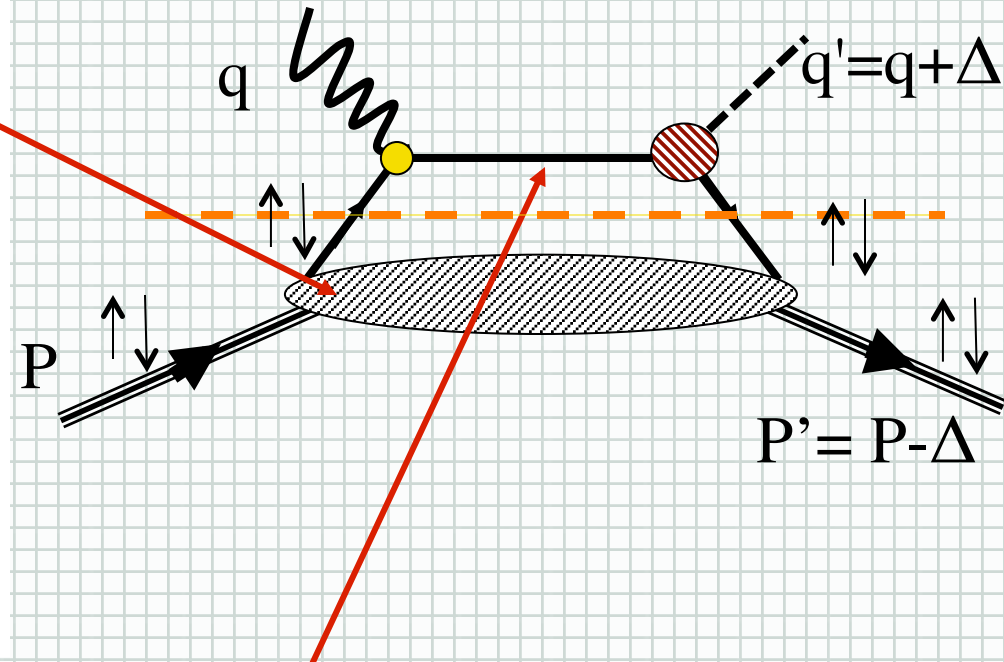
DVCS: Observables



$$f_{\Lambda_\gamma, \Delta; \Lambda_\gamma', \Delta'} = \epsilon_\mu^{\Lambda_\gamma} T_{\Lambda \Lambda'}^{\mu\nu} \epsilon_\nu^{*\Lambda_\gamma'},$$

Factorization: Quark-Proton Helicity Amplitudes

$$\begin{aligned}
 f_{++}^S &= f_{++,++} + f_{-+,-+} \\
 &= g_{++}^S \otimes (A_{++,++} + A_{-+,-+}) \\
 f_{++}^A &= f_{++,++} - f_{-+,-+} \\
 &= g_{++}^A \otimes (A_{++,++} - A_{-+,-+}) \\
 f_{+-}^S &= f_{++,+-} + f_{-+,-} \\
 &= g_{++}^S \otimes (A_{-+,++} + A_{++, -+}) \\
 f_{+-}^A &= f_{++,+-} - f_{-+,-} \\
 &= g_{++}^A \otimes (A_{-+,++} - A_{++, -+})
 \end{aligned}$$



$$g_{++}^{++} \pm g_{++}^{--} = \sqrt{X(X - \zeta)} \left(\frac{1}{X - \zeta + i\epsilon} \pm \frac{1}{X - i\epsilon} \right)$$

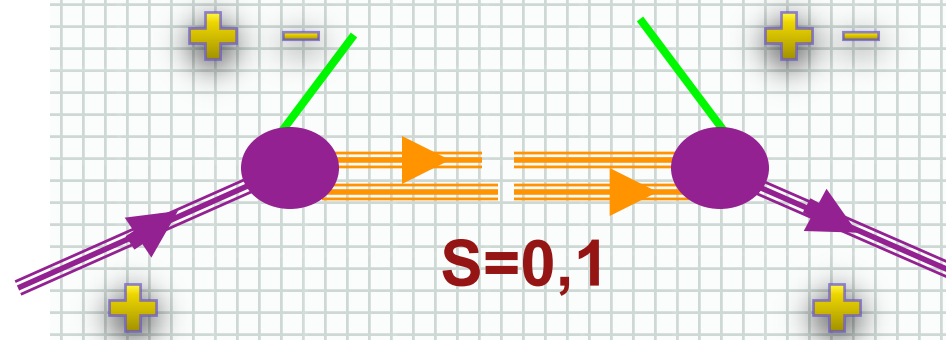
Helicity Amp. $A_{\Lambda',\lambda',\Lambda\lambda}$ in a Diquark Model

$$A_{++,++} = \int d^2k_{\perp} \phi_{++}^*(k', P') \phi_{++}(k, P)$$

$$A_{+-,+-} = \int d^2k_{\perp} \phi_{+-}^*(k', P') \phi_{+-}(k, P)$$

$$A_{-+,++} = \int d^2k_{\perp} \phi_{-+}^*(k', P') \phi_{++}(k, P)$$

$$A_{+,-+} = \int d^2k_{\perp} \phi_{++}^*(k', P') \phi_{-+}(k, P).$$



$$\phi_{++} = \mathcal{A} \bar{u}(k, +) u(P, +) = \mathcal{A} \langle k, + | P, + \rangle = \frac{\mathcal{A}}{\sqrt{X}} (m + Mx)$$

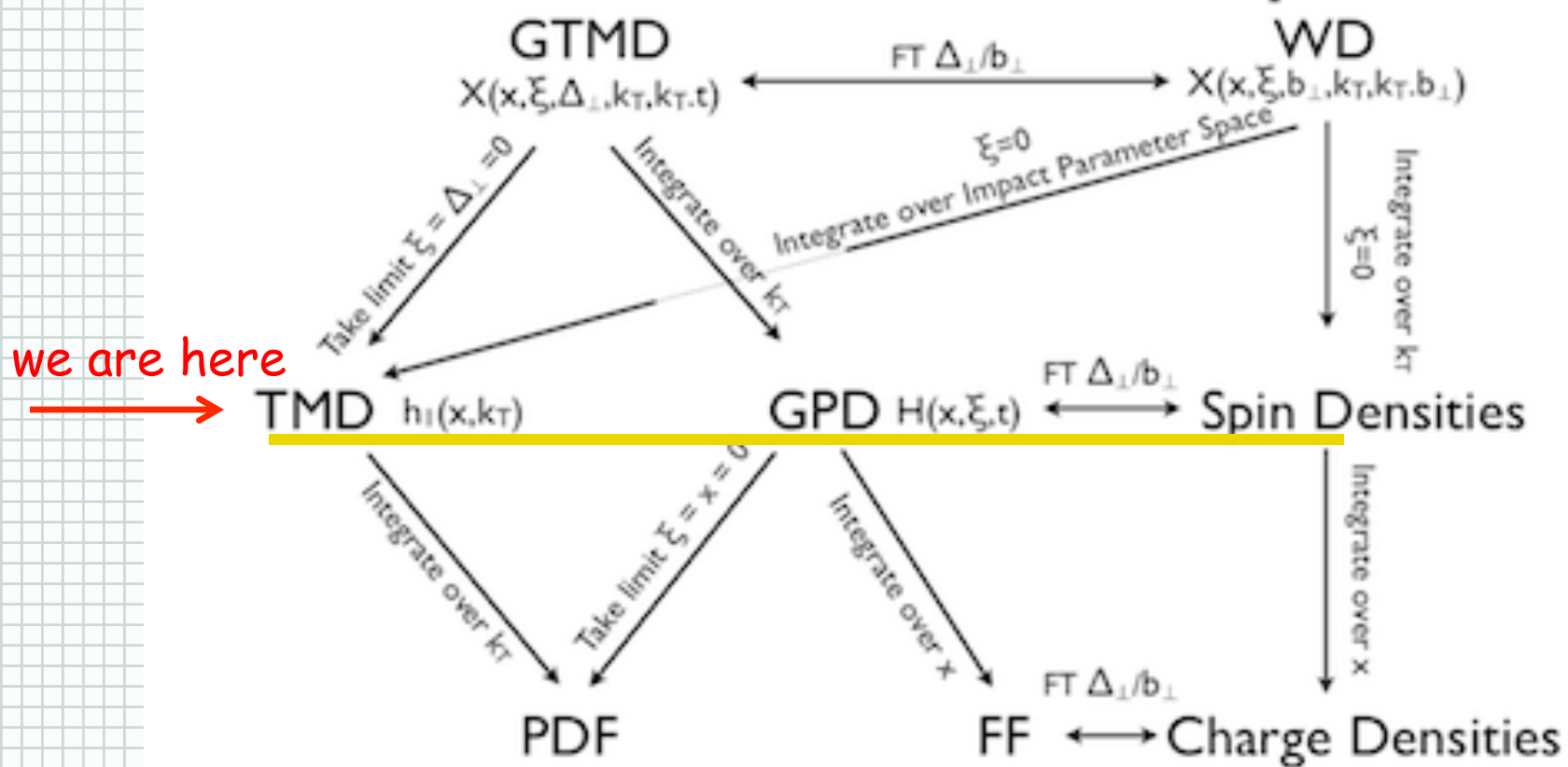
$$\phi_{--} = \mathcal{A} \bar{u}(k, -) u(P, -) = \mathcal{A} \langle k, - | P, - \rangle = \frac{\mathcal{A}}{\sqrt{X}} (m + Mx)$$

$$\phi_{+-} = \mathcal{A} \bar{u}(k, +) u(P, -) = \mathcal{A} \langle k, + | P, - \rangle = \frac{\mathcal{A}}{\sqrt{X}} (k_1 - ik_2)$$

$$\phi_{-+} = \mathcal{A} \bar{u}(k, -) u(P, +) = \mathcal{A} \langle k, - | P, + \rangle = -\frac{\mathcal{A}}{\sqrt{X}} (k_1 + ik_2)$$

What happens at the unintegrated level

Distribution Graph



M. Murray

Connection between "cartesian" and helicity bases

GTMDs (Metz et al, 2009)

$$W_{\lambda\lambda'}^{[\gamma^+]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1} + \frac{i\sigma^{i+} k_T^i}{P^+} F_{1,2} + \frac{i\sigma^{i+} \Delta_T^i}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_T^i \Delta_T^j}{M^2} F_{1,4} \right] u(p, \lambda),$$

$$W_{\lambda\lambda'}^{[\gamma^+ \gamma_5]} = \frac{1}{2M} \bar{u}(p', \lambda') \left[-\frac{i\varepsilon_T^{ij} k_T^i \Delta_T^j}{M^2} G_{1,1} + \frac{i\sigma^{i+} \gamma_5 k_T^i}{P^+} G_{1,2} + \frac{i\sigma^{i+} \gamma_5 \Delta_T^i}{P^+} G_{1,3} + i\sigma^{+-} \gamma_5 G_{1,4} \right] u(p, \lambda),$$

GPDs

$$A_{++;++} = \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left(\frac{H + \tilde{H}}{2} - \frac{\zeta^2/4}{1-\zeta} \frac{E + \tilde{E}}{2} \right)$$

$$A_{+-;+-} = \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left(\frac{H - \tilde{H}}{2} - \frac{\zeta^2/4}{1-\zeta} \frac{E - \tilde{E}}{2} \right)$$

$$A_{++;-+} = -\frac{\Delta_1 - i\Delta_2}{2M} \left[E - \frac{\zeta/2}{1-\zeta/2} \tilde{E} \right]$$

$$A_{-+;++} = \frac{\Delta_1 + i\Delta_2}{2M} \left[E + \frac{\zeta/2}{1-\zeta/2} \tilde{E} \right]$$

These objects are at least twice as many and complex

Need to define a way of counting the number of independent objects

For form factors & gravitomagnetic form factors use J^{PC} quantum numbers of crossed channel – $NN\bar{}$ states (Z.Chen and Ji, Hagler, GGL 2012)
(next slide)

Here we establish the criterion that we keep only the combinations of helicity amplitudes that are diagonal in a given spin basis: we are lead by the physical interpretation (physical quantities exist only if they can be tied to an observable)

By using Parity constraints on the Amps. we find a smaller number of GTMDs than in Metz et al.

General rule to count form factors: t-channel J^{PC} q. numbers

n	$J^{PC}(S; L)$					
0	0^{+-}	$1^{--}(1; 0, 2)$				
1	$0^{++}(1; 1)$	1^{-+}	$2^{++}(1; 1, 3)$			
2	0^{+-}	$1^{--}(1; 0, 2)$	2^{+-}	$3^{--}(1; 2, 4)$		
3	$0^{++}(1; 1)$	1^{-+}	$2^{++}(1; 1, 3)$	3^{-+}	$4^{++}(1; 3, 5)$	
...	...					

Haegler, PLB(2004)
Z.Chen&Ji, PRD(2005)

TABLE III: J^{PC} of the vector operators with $(S; L, L')$ for the corresponding $N\bar{N}$ state. Where there are no $(S; L, L')$ values there are no matching quantum numbers for the $N\bar{N}$ system.

Nucleon

	$L=0$	1	2	3	4	...
$S=0$	$J^{PC} 0^{-+}$	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
$S=1$	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Deuteron

	$L=0$	1	2	3	4	...
$S=0$	$J^{PC} 0^{++}$	1^{--}	2^{++}	3^{--}	4^{++}	
$S=1$	1^{+-}	0^{+-}	1^{+-}	2^{-+}	3^{+-}	
		1^{-+}	2^{+-}	3^{-+}	4^{+-}	
		2^{-+}	3^{+-}	4^{-+}	5^{+-}	
$S=2$	2^{++}	1^{--}	0^{++}	1^{--}	2^{++}	
		2^{--}	1^{++}	2^{--}	3^{++}	
		3^{--}	2^{++}	3^{--}	4^{++}	
			3^{++}	4^{--}	5^{++}	
			4^{++}	5^{--}	6^{++}	

TABLE II: J^{PC} of the $d\bar{d}$ states.

Both S and L states considered (related to X.Ji's talk)

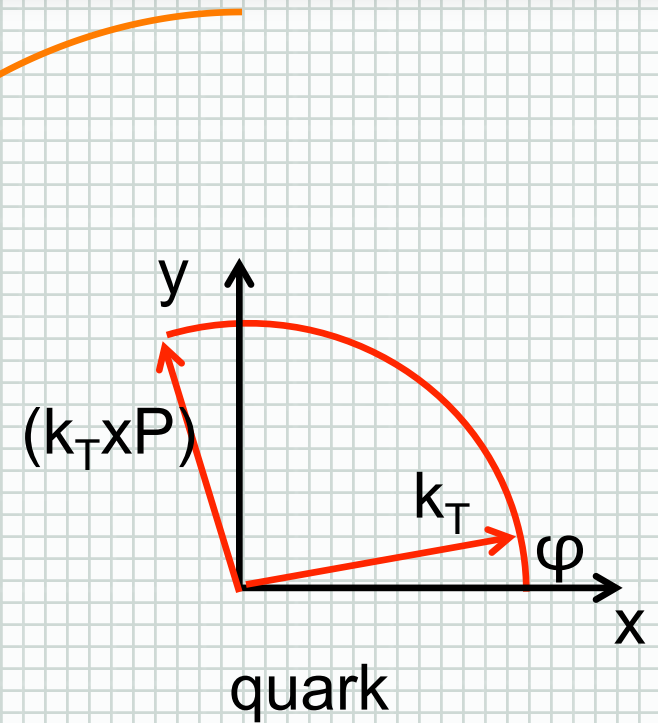
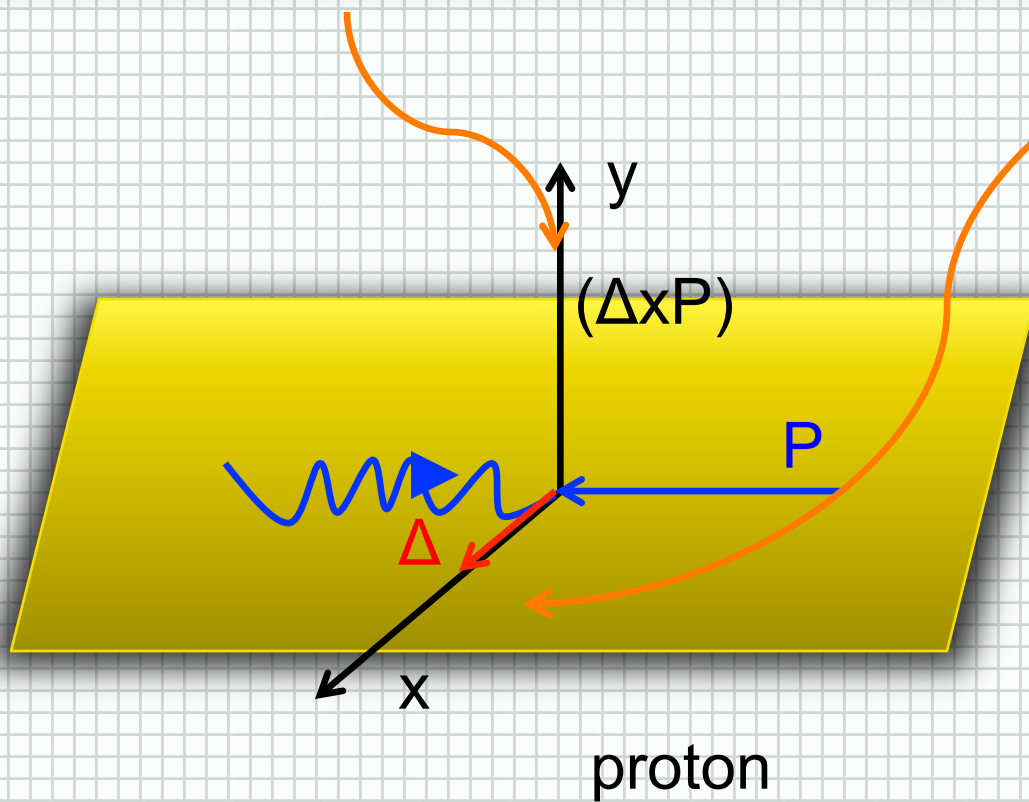
Transversity bases

Canonical

$$|P, +(-)\rangle^{Ty} = \frac{1}{\sqrt{2}} [|P, +\rangle + (-)i |P, -\rangle]$$

Planar

$$|P, +(-)\rangle^{Tx} = \frac{1}{\sqrt{2}} [|P, +\rangle + (-) |P, -\rangle]$$



transversity state

final

initial

A^T x Λ' λ' , $\Lambda\lambda$

proton

quark

Chiral Even Summary

L

$$2\tilde{H}(X, k_{\perp}, 0, \Delta_{\perp}) = A_{+,+,++} - A_{+,-,+} + A_{-,-,-} - A_{-+,-}$$

T_Y

$$-\frac{\Delta_{\perp}}{M} E(X, k_{\perp}, 0, \Delta_{\perp}) = -i(A_{+,+,++}^{T_Y} + A_{+,-,+}^{T_Y} - A_{-+,-}^{T_Y} - A_{-,-,-}^{T_Y})$$

T_X

$$\xi \frac{\Delta_{\perp}}{M} \tilde{E}(X, k_{\perp}, 0, \Delta_{\perp}) = (A_{+,+,++}^{L,T_Y} + A_{+,-,+}^{L,T_Y} - A_{-+,-}^{L,T_Y} - A_{-,-,-}^{L,T_Y})$$

Chiral Odd Summary

T_Y

$$\tau \left[2\tilde{H}_T(X, k_{\perp}, 0, \Delta_{\perp}) + E_T(X, 0, t) \right] = A_{+,+,++}^{T_Y} - A_{+,-,+}^{T_Y} + A_{-+,-}^{T_Y} - A_{-,-,-}^{T_Y}$$

T_X

$$H_T(X, k_{\perp}, 0, \Delta_{\perp}) = A_{+,+,++}^{T_X} - A_{+,-,+}^{T_X} - A_{-+,-}^{T_X} + A_{-,-,-}^{T_X}$$

T_{Y,X}

$$\tau^2 \tilde{H}_T(X, k_{\perp}, 0, \Delta_{\perp}) = A_{+,+,++}^{T_Y} - A_{+,-,+}^{T_Y} - A_{-+,-}^{T_X} + A_{-,-,-}^{T_X}$$

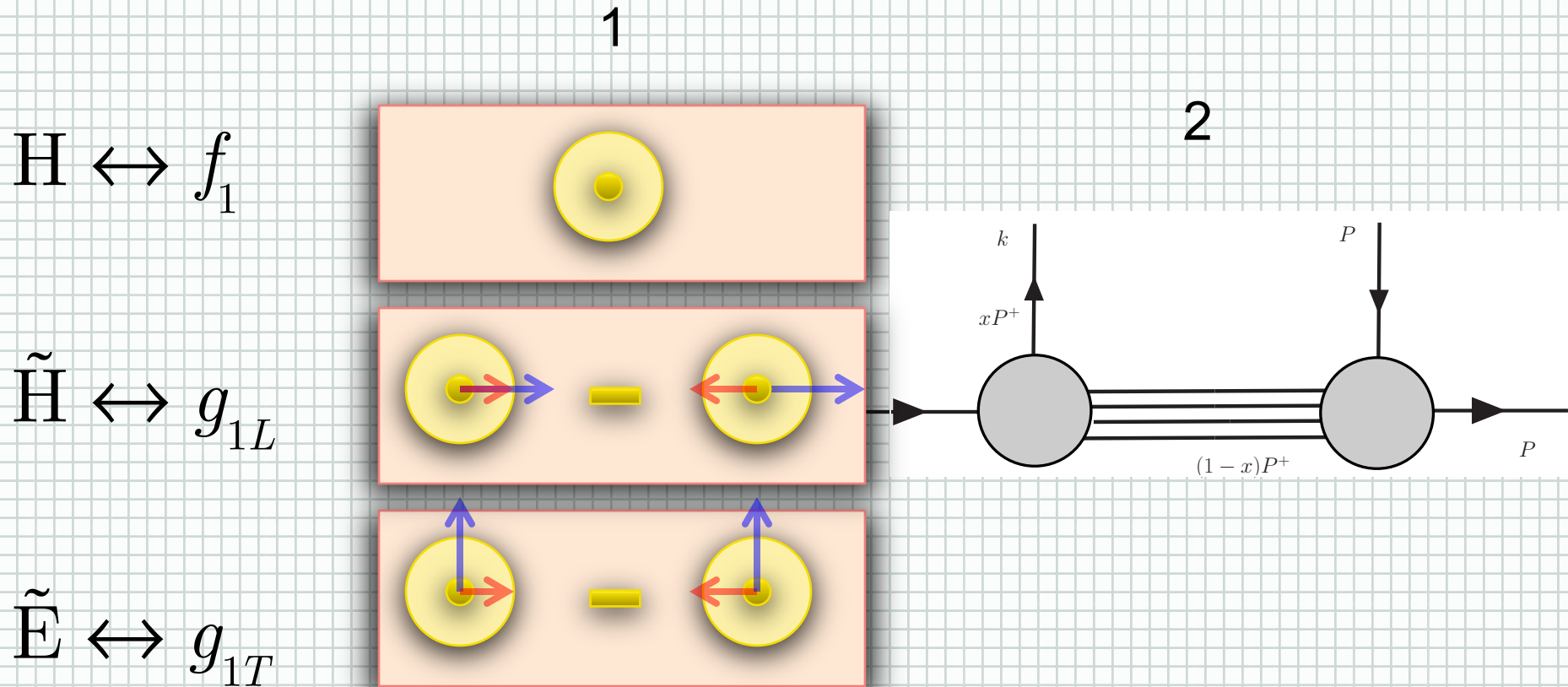
T_Y

$$\left[H_T(X, k_{\perp}, 0, \Delta_{\perp}) + \frac{1}{2} \tau^2 \tilde{H}_T(X, k_{\perp}, 0, \Delta_{\perp}) \right] = A_{+,+,++}^{T_Y} - A_{+,-,+}^{T_Y} - A_{-+,-}^{T_Y} + A_{-,-,-}^{T_Y}$$

... on to multi-parton distributions

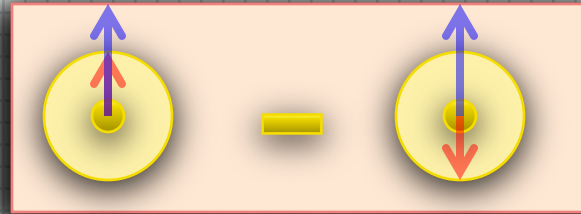
Chiral Even Sector \rightarrow GPDs and TMDs with:

1. same helicity/transversity structure
2. Same parton correlations in a suitably defined forward limit

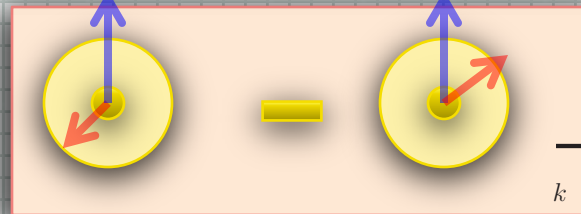


Chiral Odd Sector

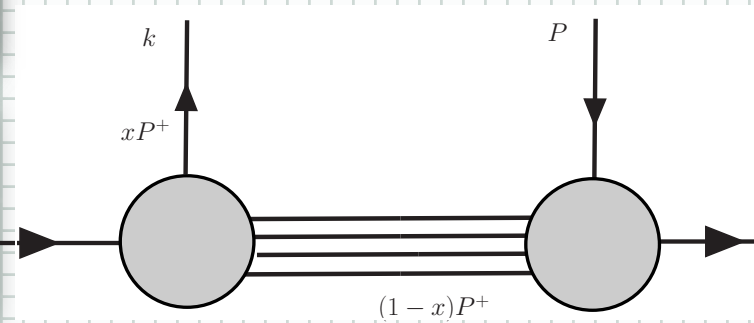
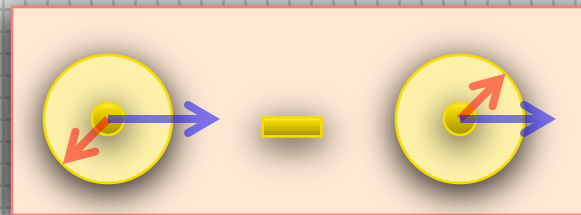
$$H_T \leftrightarrow h_1$$



$$-\tau^2 \tilde{H}_T \leftrightarrow h_{1T}^\perp$$

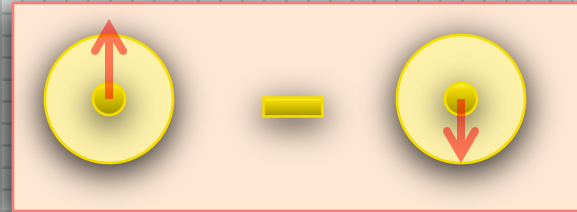


$$\tau \tilde{E}_T \leftrightarrow h_{1L}^\perp$$

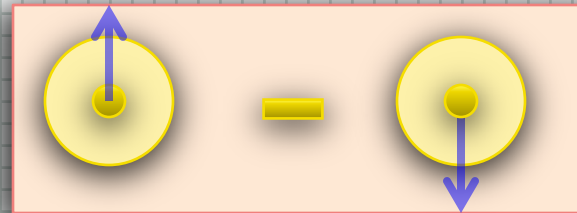


SSA Sector: GPDs and TMDs with same helicity/transversity structure

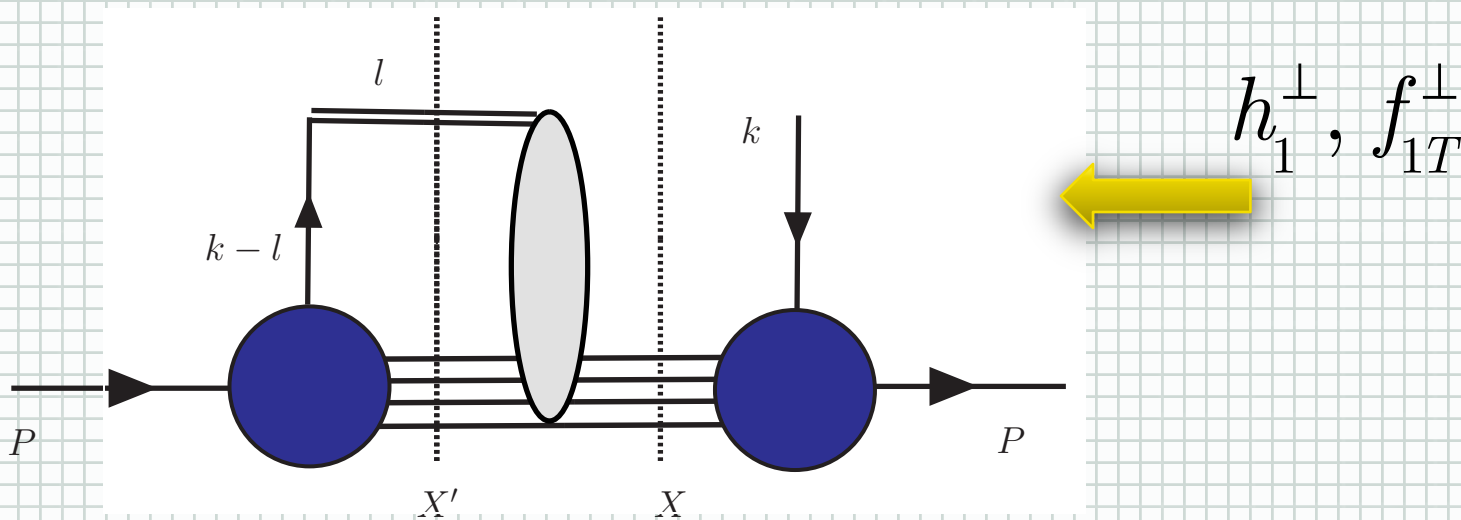
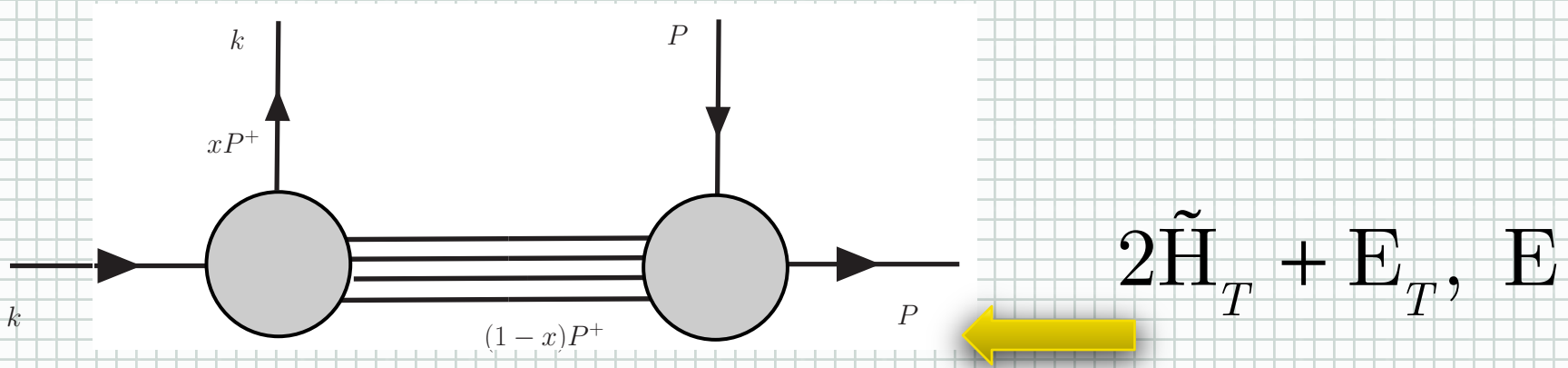
$$2\tilde{H}_T + E_T \leftrightarrow h_1^\perp$$

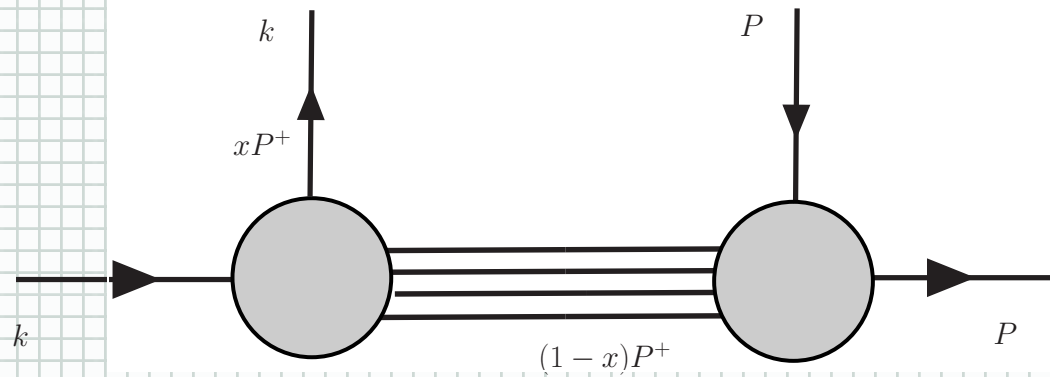


$$E \leftrightarrow f_{1T}^\perp$$



...but different multi-parton correlations in a suitably defined forward limit (aside from spin for a moment...)





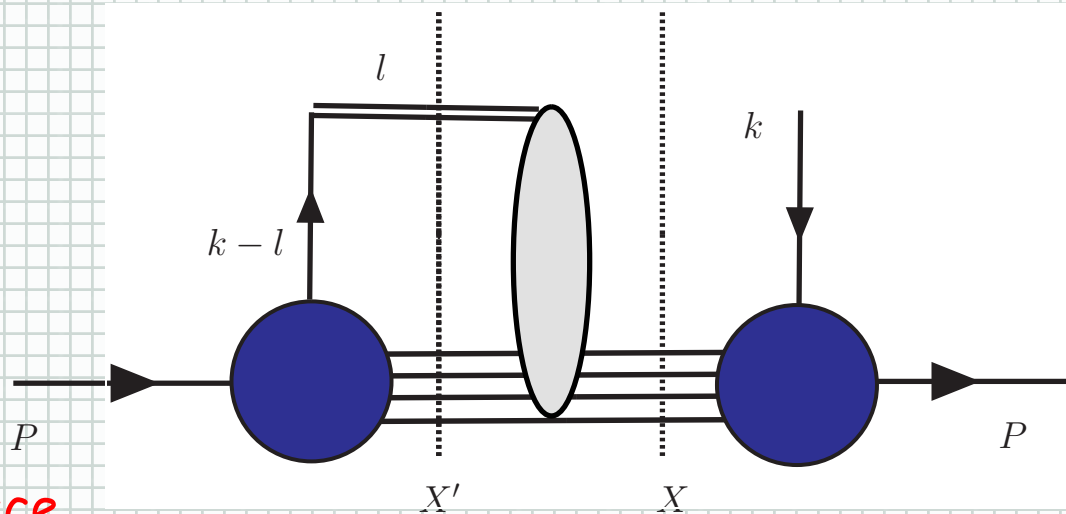
Momentum Space: diagonal in x non diagonal in k_T

$$\begin{aligned}
 E(x, 0, t) &= \int dz^- \exp^{ixP^+z^-} \langle P' | \bar{\psi}(0, z^-, 0_T) \Gamma \psi(0, 0, 0_T) | P \rangle = \\
 &= \sum_X \int dz^- \exp^{i(xP^+ - P'^+ + P_X^+)z^-} \langle P' | \bar{\psi}_+(0) | X \rangle \langle X | \psi_+(0, 0, 0_T) | P \rangle = \\
 &= \int d^2k_T \phi^*(x, \underline{k_T - \Delta}) \phi(x, \underline{k_T})
 \end{aligned}$$

Coordinate Space: diagonal in b

$$E(x, 0, t) = \int d^2b e^{ib \cdot \Delta} \rho(x, \underline{b}) \quad t = \Delta^2$$

GPDs and non SSA TMDs are one particle density distributions!



Momentum Space

$$\begin{aligned}
 \langle k_T^i(x) \rangle_{UT} &= \int dz^- \exp^{ixP^+z^-} \langle P | \bar{\psi}(0, -z/2, 0_T) \gamma^+ [\mathcal{W}(-z/2, z/2) I_q(z/2)] \psi(0, z/2, 0_T) | P \rangle = \\
 &= \sum_{X, X'} \int dz^- \exp^{i(xP^+ - P^+ + P_X'^+)z^-} \langle P | \bar{\psi}_+(0) | X \rangle \langle X' | \psi_+(0) | P \rangle \langle X | \mathcal{W}(0, 0) I_q(0) | X' \rangle = \\
 &= \int d^2 k_T \int d^2 l_T \phi^*(x, k_T) \phi(x, k_T - l_T, P) L(x, l_T)
 \end{aligned}$$

FSI

Coordinate Space

But careful! Although the correlator factorizes into a GTMD and FSI, it describes multiparton correlations which are different from the TMDs

$$\langle k_T^i(x) \rangle_{UT} = \int d^2b_1 d^2b'_1 d^2b_2 \rho_2[(x, b_1), (0, b_2); (x, b'_1), (0, b_2)] I(b_1 - b_2)$$

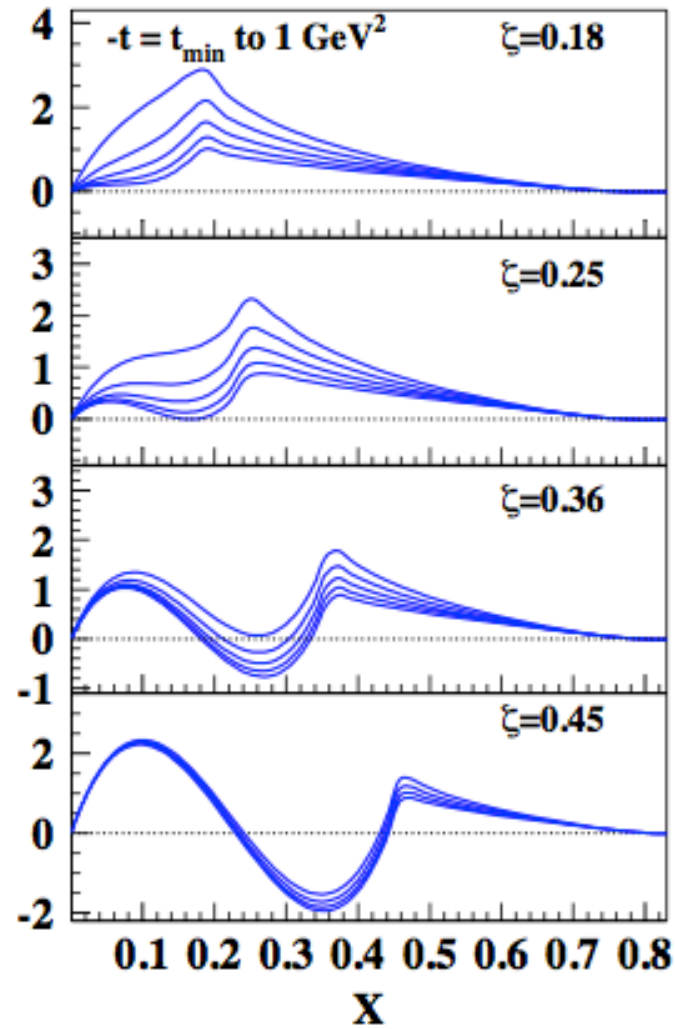
semi-diagonal (in b) two-particle density distribution

The multiparticle densities scenario provides a necessary theoretical/formal context/background.

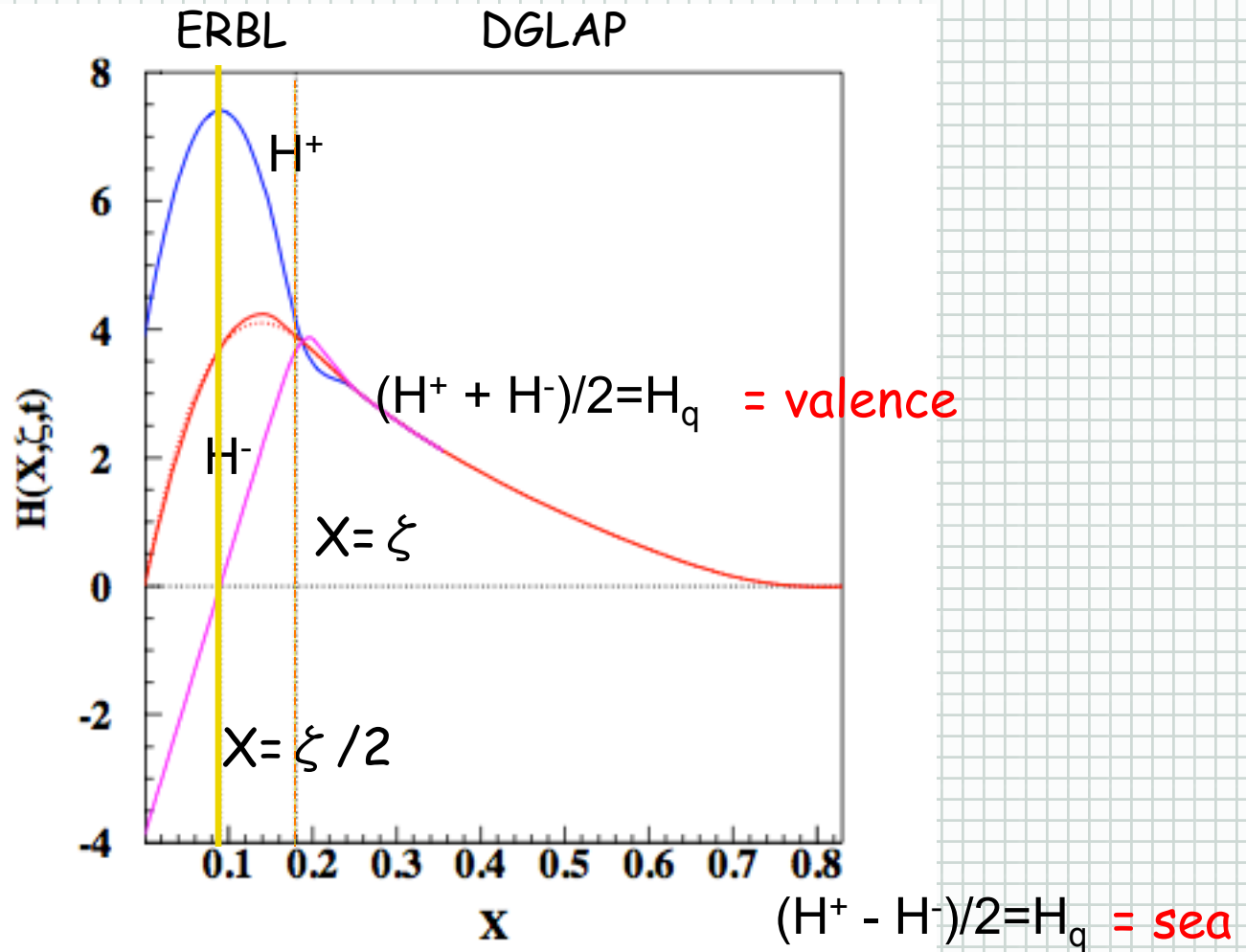
- We understand what makes diquark and quark target models “simple” in this context → they are two component models, therefore all of the “complexity” of multiparton interactions is glossed over, FSI is a simple multiplicative factor.
- More realistic diquark type models (with S and D wave spectators, see e.g. Goldstein and Liebl, PL1995; F.Gross and T. Peña, PRD 2011, or taking into account the internal momenta of the spectators, work in progress) could in principle give a very different answer
- Quark models could in principle give a very different answer (see e.g. A. Courtoy and S. Scopetta, PRD 2009)

...on to physically motivated parametrization of data ...

$$H_q(X, \zeta, t)$$



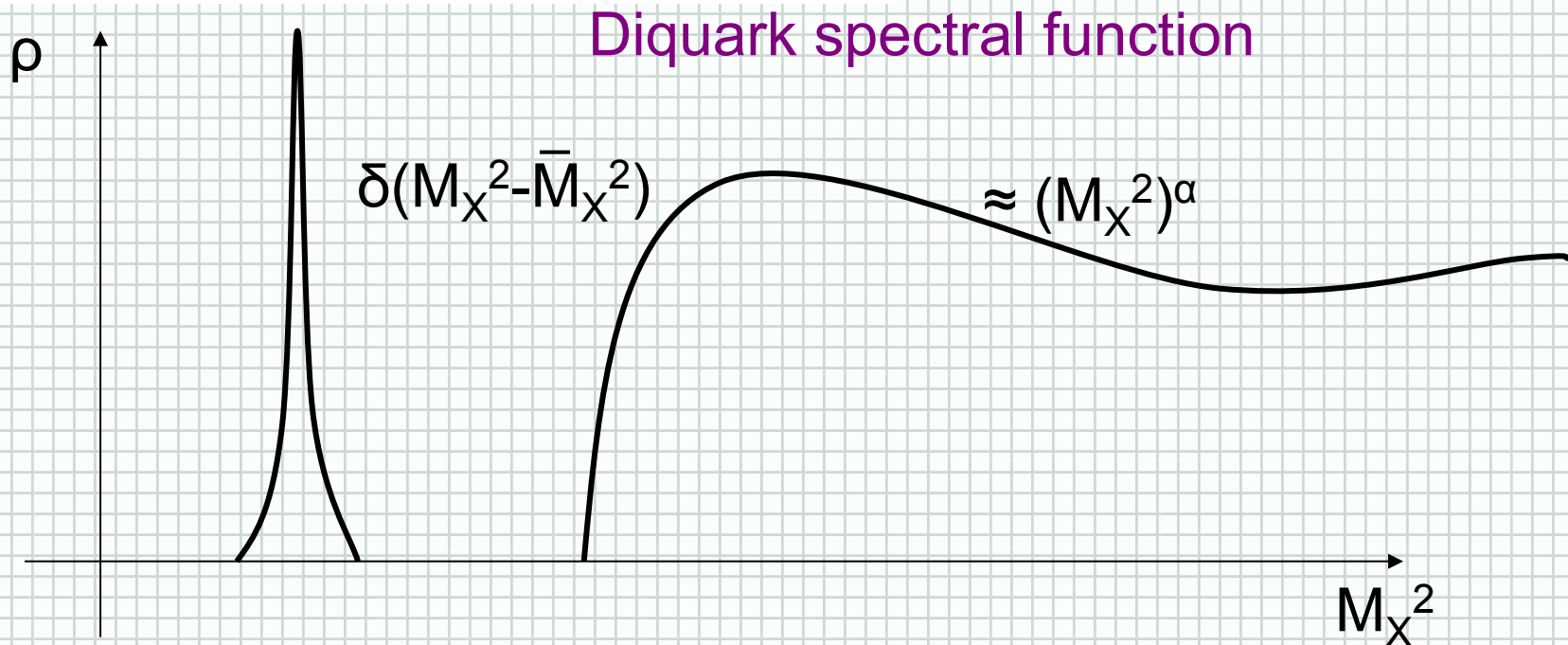
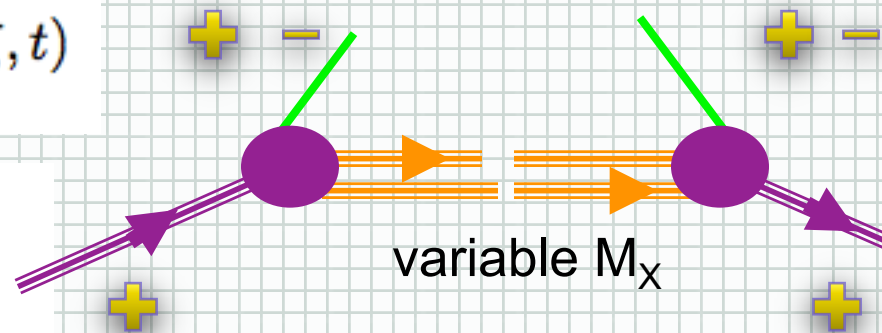
Crossing Symmetries



Parametric Form: Reggeized Diquark Model

$$F(X, \zeta, t) = \mathcal{N} G_{M_X, m}^{M_\Lambda} (X, \zeta, t) R_p^{\alpha, \alpha'} (X, \zeta, t)$$

$$\int_0^\infty dM_X^2 \rho_R(M_X^2) H(X, 0, 0) \sim X^{-\alpha(0)-1},$$



Brodsky, Close, Gunion → DIS ('70s)

Gorshteyn & Szczepaniak (PRD, 2010)

Brodsky, Llanes, Szczepaniak arXiv:0812.0395

$$F(X, \zeta, t) = \mathcal{N} G_{M_X, m}^{M_\Lambda}(X, \zeta, t) R_p^{\alpha, \alpha'}(X, \zeta, t)$$

$$H = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_\perp \frac{\left[(m + MX) \left(m + M \frac{X - \zeta}{1 - \zeta} \right) + \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp \right]}{(k^2 - M_\Lambda^2)^2 (k'^2 - M_\Lambda^2)^2} + \frac{\zeta^2}{4(1 - \zeta)} E,$$

$$E = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_\perp \frac{-2M(1 - \zeta) \left[(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_\perp^2} - \left(m + M \frac{X - \zeta}{1 - \zeta} \right) \frac{k_\perp \cdot \Delta}{\Delta_\perp^2} \right]}{(k^2 - M_\Lambda^2)^2 (k'^2 - M_\Lambda^2)^2}$$

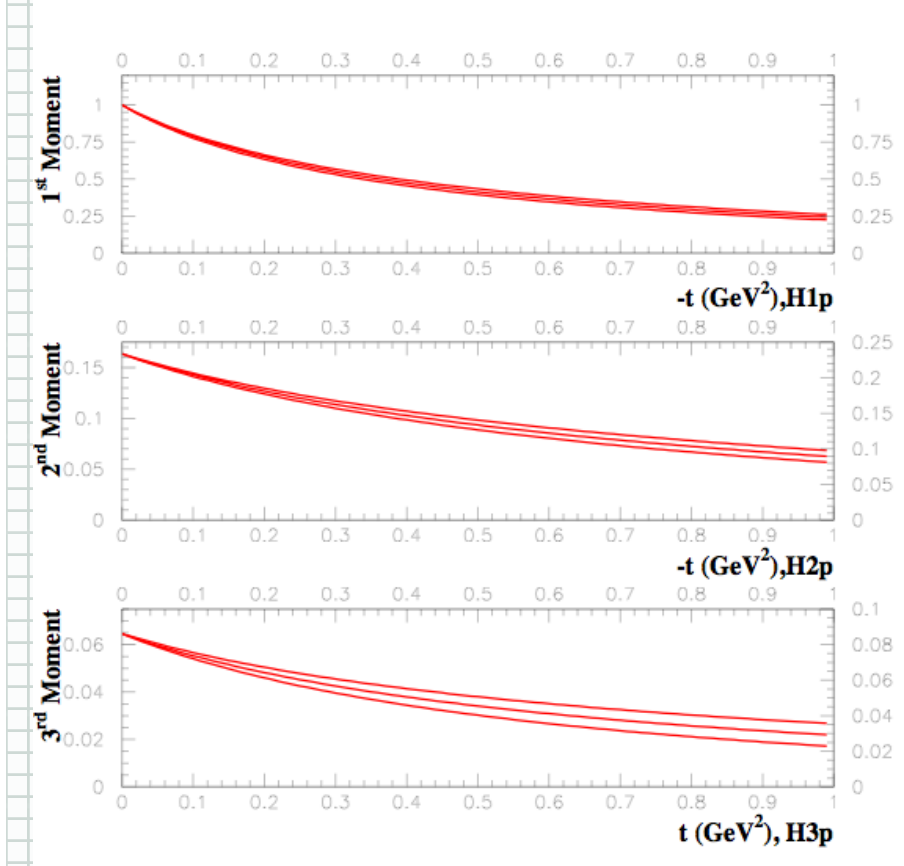
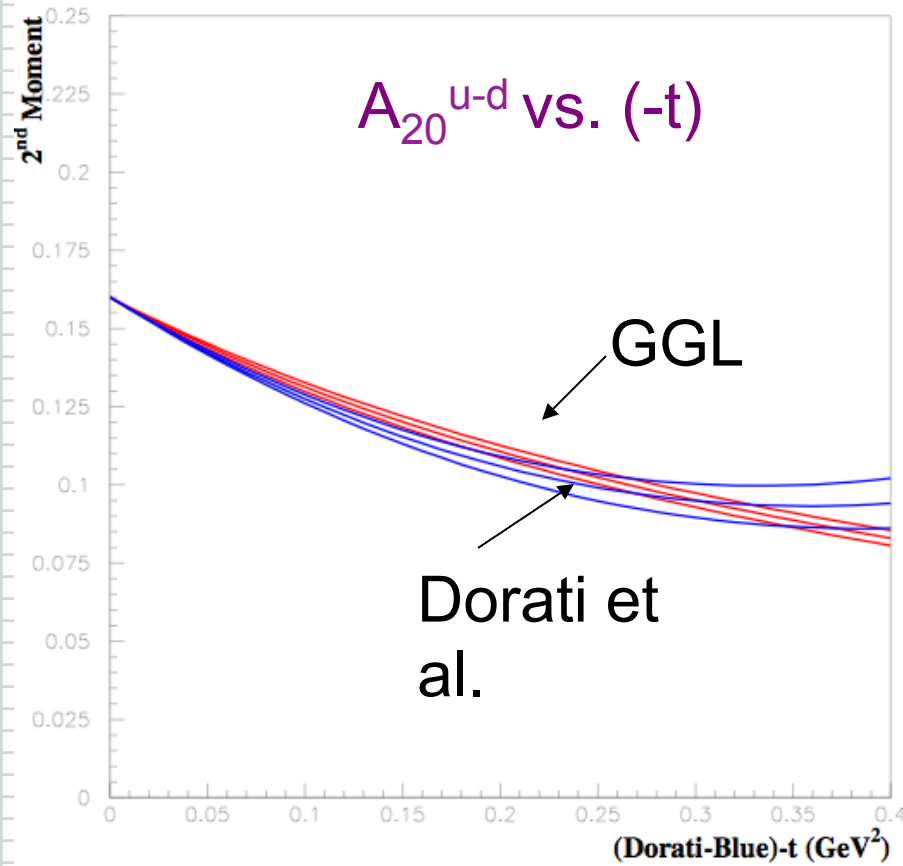
$$\tilde{H} = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_\perp \frac{\left[(m + MX) \left(m + M \frac{X - \zeta}{1 - \zeta} \right) - \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp \right]}{(k^2 - M_\Lambda^2)^2 (k'^2 - M_\Lambda^2)^2} + \frac{\zeta^2}{4(1 - \zeta)} \tilde{E}$$

$$\tilde{E} = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_\perp \frac{-\frac{4M(1 - \zeta)}{\zeta} \left[(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_\perp^2} + \left(m + M \frac{X - \zeta}{1 - \zeta} \right) \frac{k_\perp \cdot \Delta}{\Delta_\perp^2} \right]}{(k^2 - M_\Lambda^2)^2 (k'^2 - M_\Lambda^2)^2}$$

We asked the question: "What is the minimal number of parameters necessary to fit X and t ?" Can be addressed with Recursive Fit

Parameters	H	E	\tilde{H}	\tilde{E}
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	2.448 ± 0.0885	2.811 ± 0.765	1.543 ± 0.296	5.130 ± 0.101
p_u	0.620 ± 0.0725	0.863 ± 0.482	0.346 ± 0.248	3.507 ± 0.054
\mathcal{N}_u	2.043	1.803	0.0504	1.074
χ^2	0.773	0.664	0.116	1.98
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	2.209 ± 0.156	1.362 ± 0.585	1.298 ± 0.245	3.385 ± 0.145
p_d	0.658 ± 0.257	1.115 ± 1.150	0.974 ± 0.358	2.326 ± 0.137
\mathcal{N}_d	1.570	-2.800	-0.0262	-0.966
χ^2	0.822	0.688	0.110	1.00

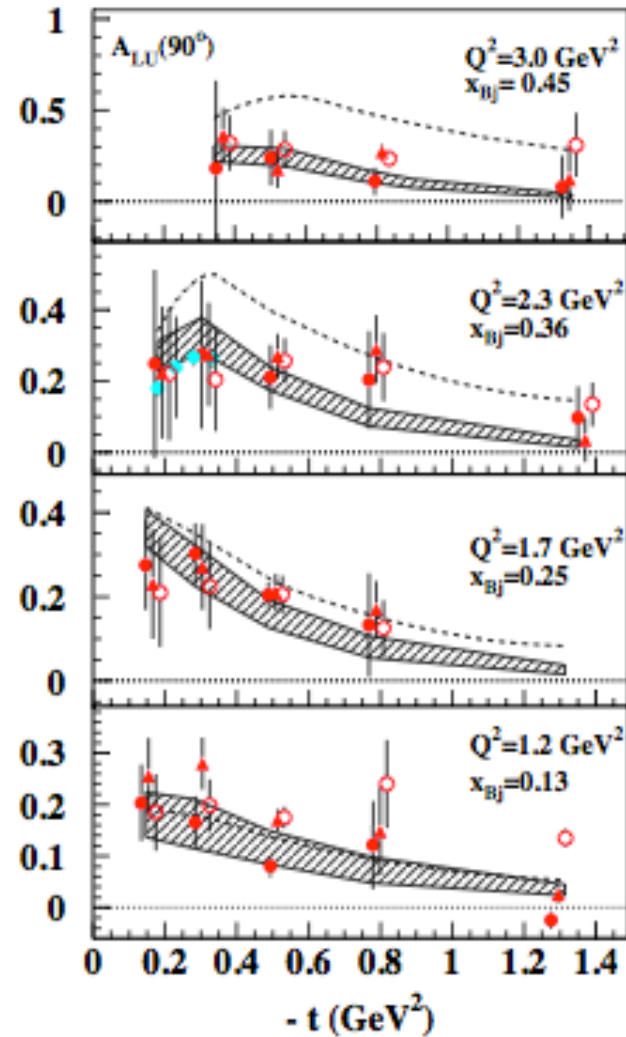
Comparison with lattice



Implementing DVCS data...

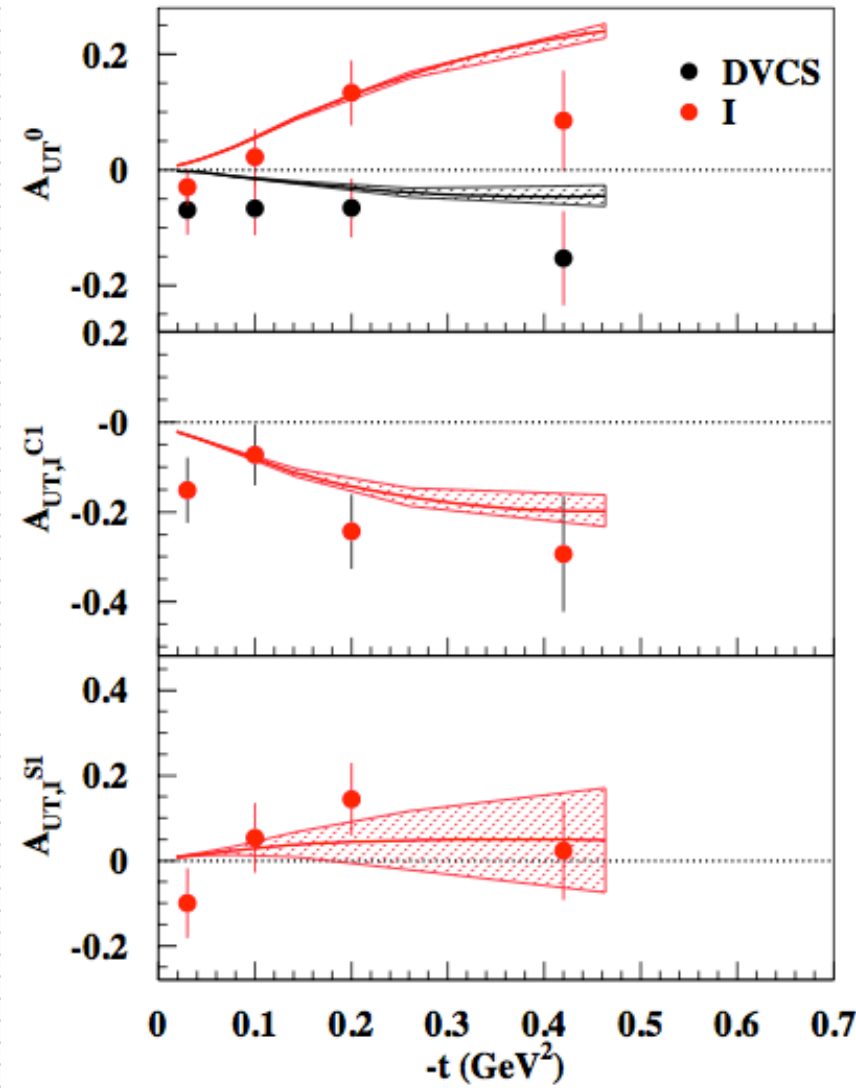
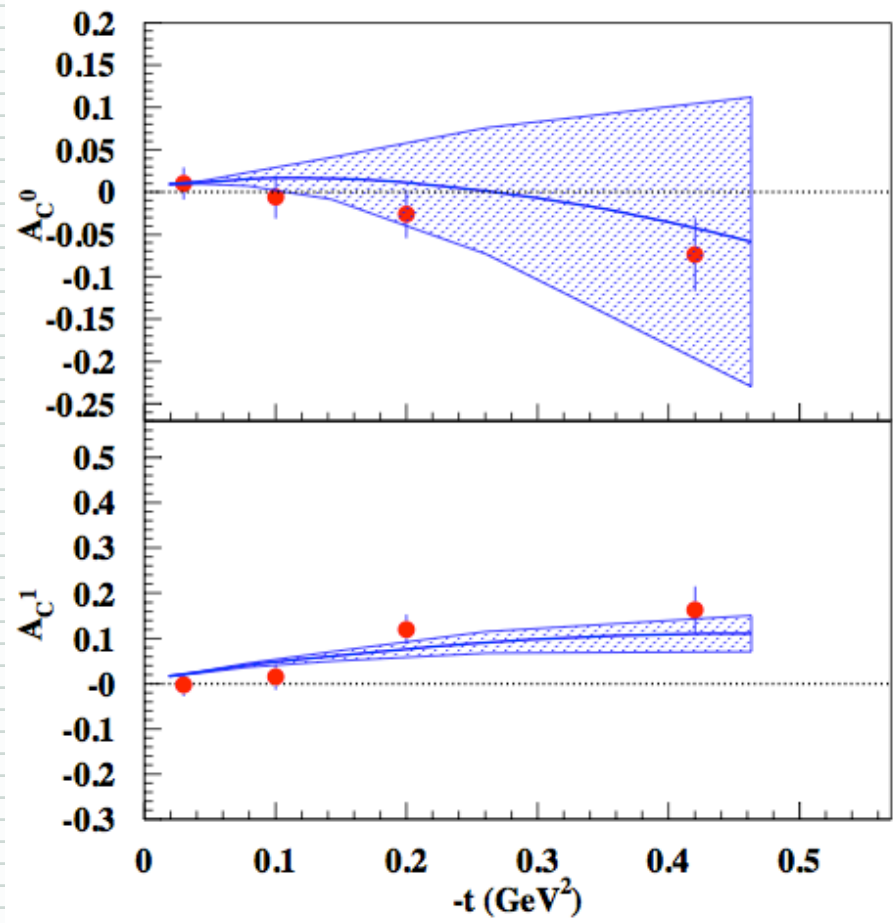
$$R = X^{-[\alpha + \alpha'(X)t + \beta(\zeta)t]},$$

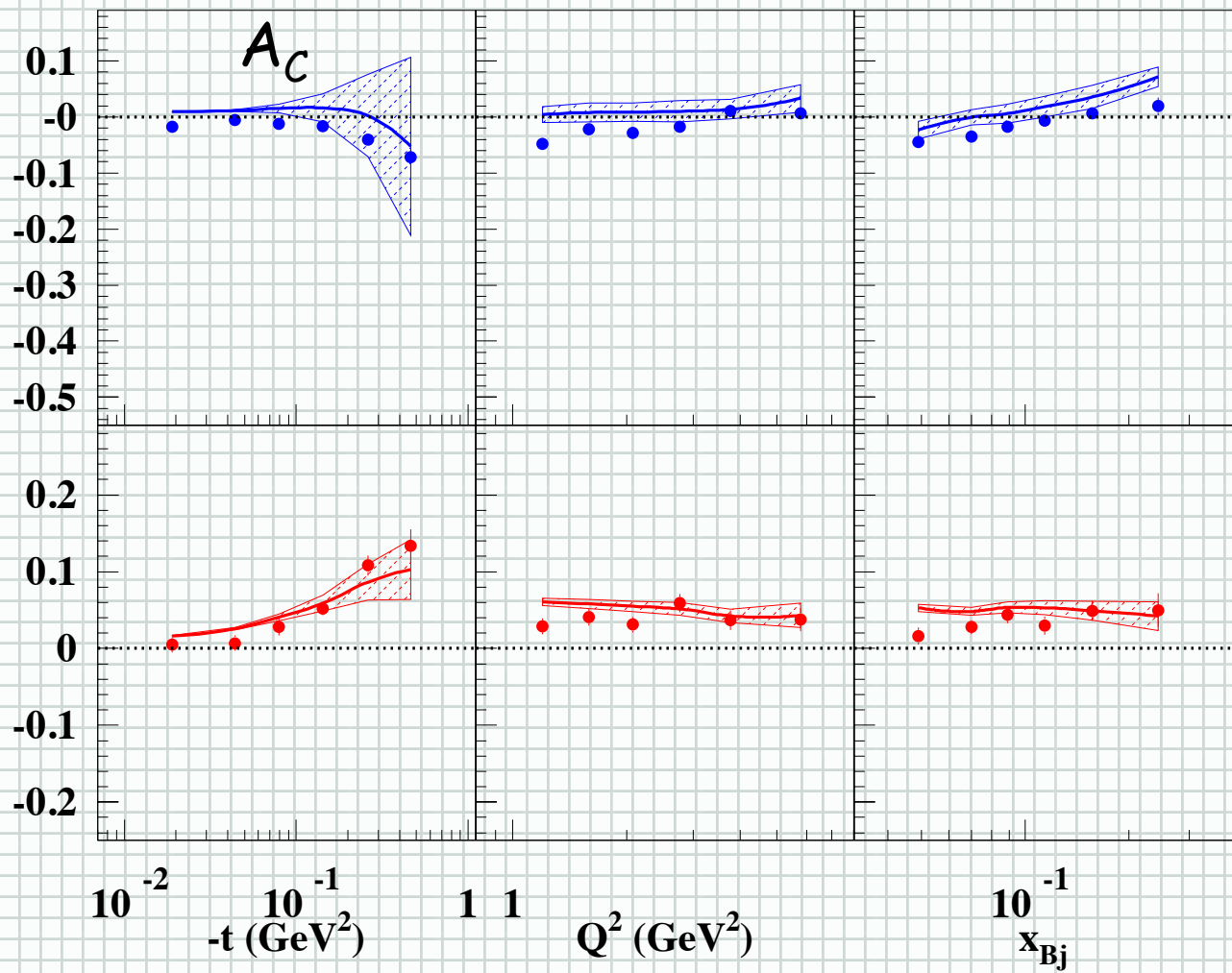
extra term



Having fitted Jlab data, we predict Hermes

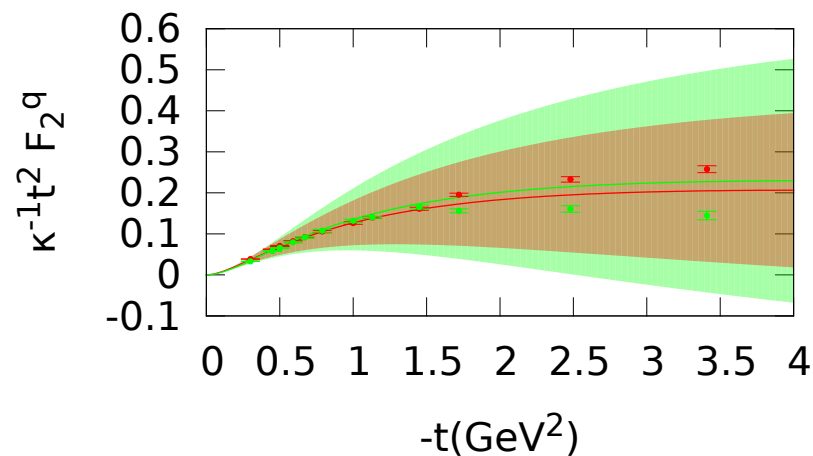
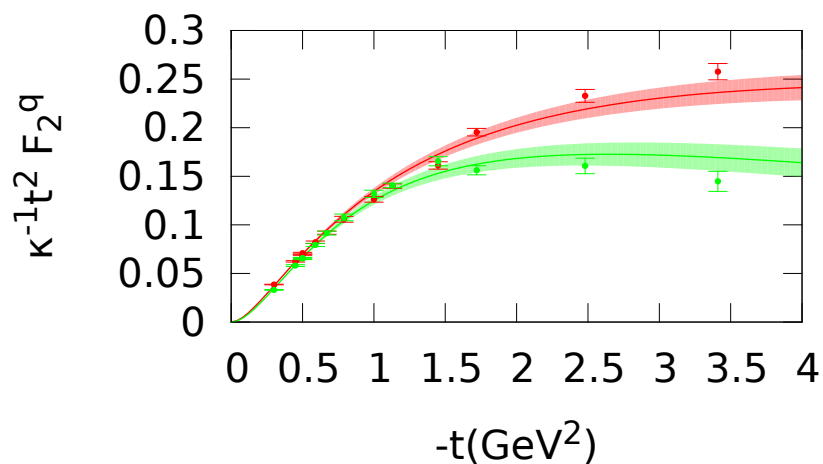
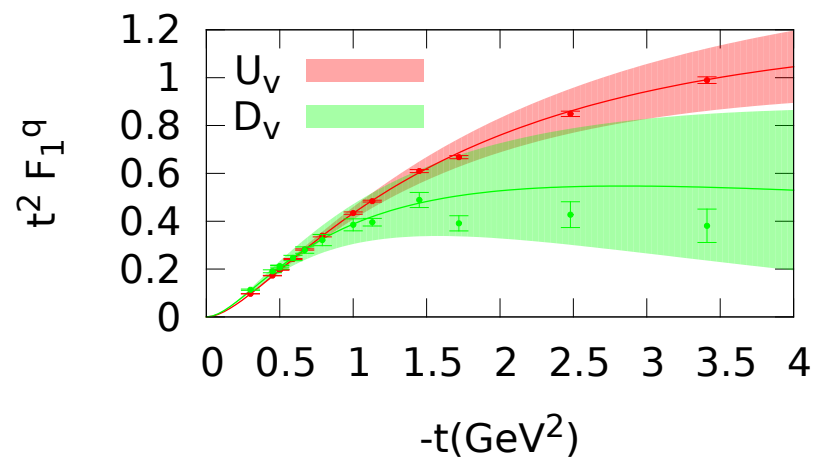
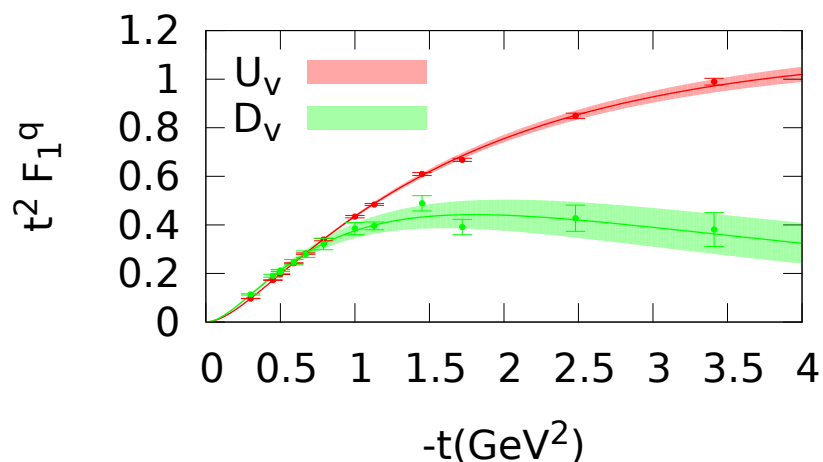
Goldstein et al. arXiv:1012.3776





New flavor separated data on form factors (G.Cates et al., PRL 2011)

Interpretation? We believe it is not a simple model (pointlike diquark excluded), but can explain it in terms of reggeized diquark model



After

...Before

Chiral Odd Sector

Using Parity relations with scalar (S=0) and axial vector (S=1) diquarks we can predict all GPDs

$S = 0$

$$\tilde{H}_T^{(0)} = -\frac{M(1-x)}{m+Mx} E^{(0)}$$

$$E_T^{(0)} = 2 \left(1 + \frac{M(1-x)}{m+Mx} \right) E^{(0)}$$

$$\tilde{E}_T^{(0)} = 0$$

$$H_T^{(0)} = \frac{H^{(0)} + \tilde{H}^{(0)}}{2} - \frac{t_0 - t}{4M^2} \frac{M(1-x)}{m+Mx} E^{(0)}$$

$S = 1$

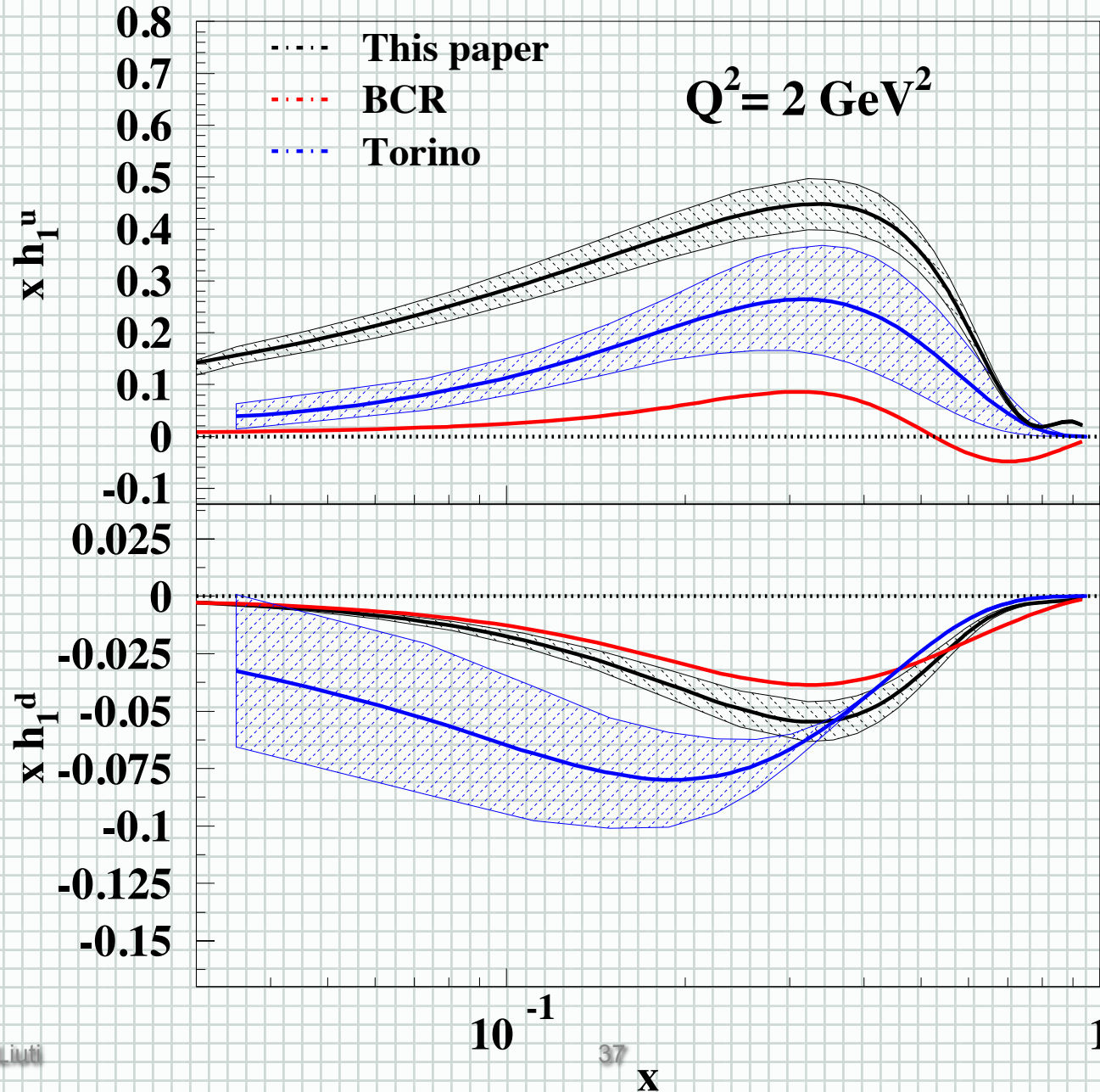
$$\tilde{H}_T^{(1)} = 0$$

$$E_T^{(1)} = 2E^{(1)}$$

$$\tilde{E}_T^{(1)} = 0$$

$$H_T^{(1)} = -\frac{2x}{1+x^2} \frac{H^{(1)} + \tilde{H}^{(1)}}{2}$$

Extraction of transversity using DVCS data



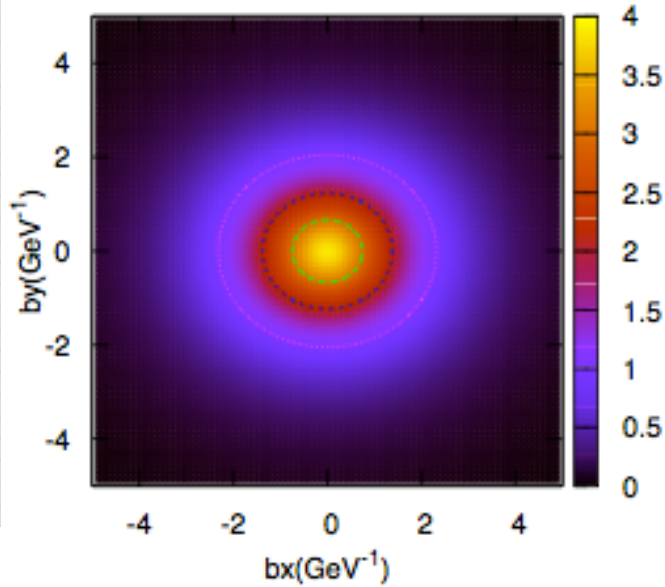
Wigner distribution studies

Gonzalez, Goldstein, S.L., R-Diquark Model

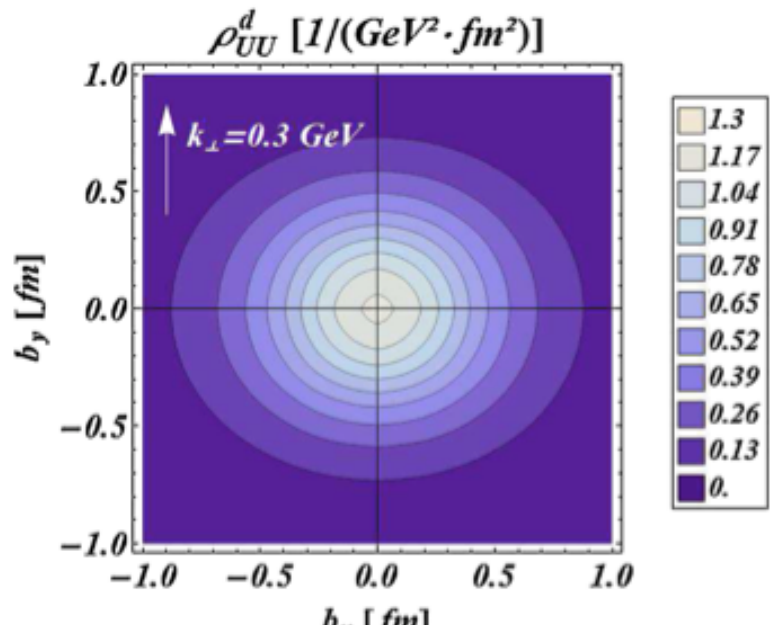
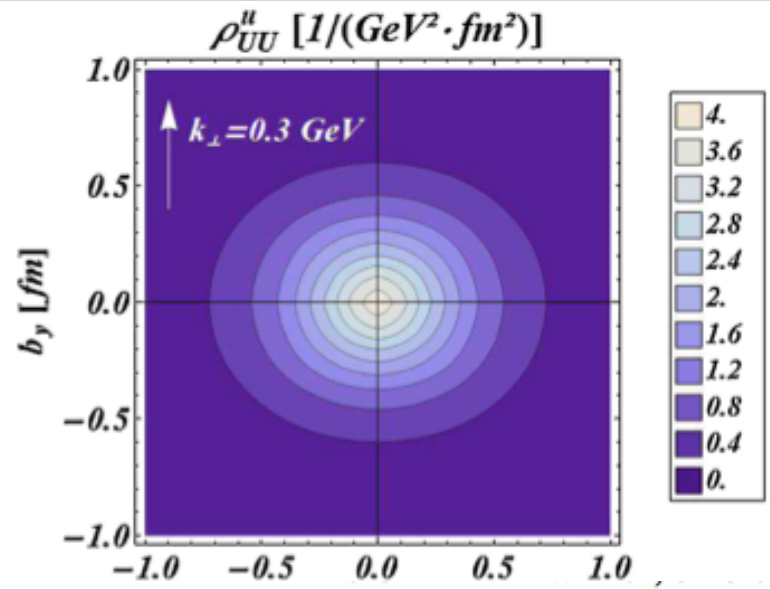
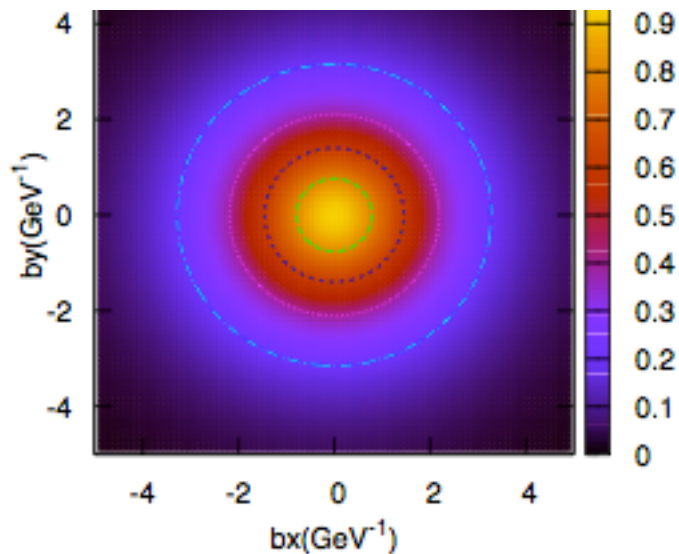
Lorce, Pasquini, (2011) LCCQM

H

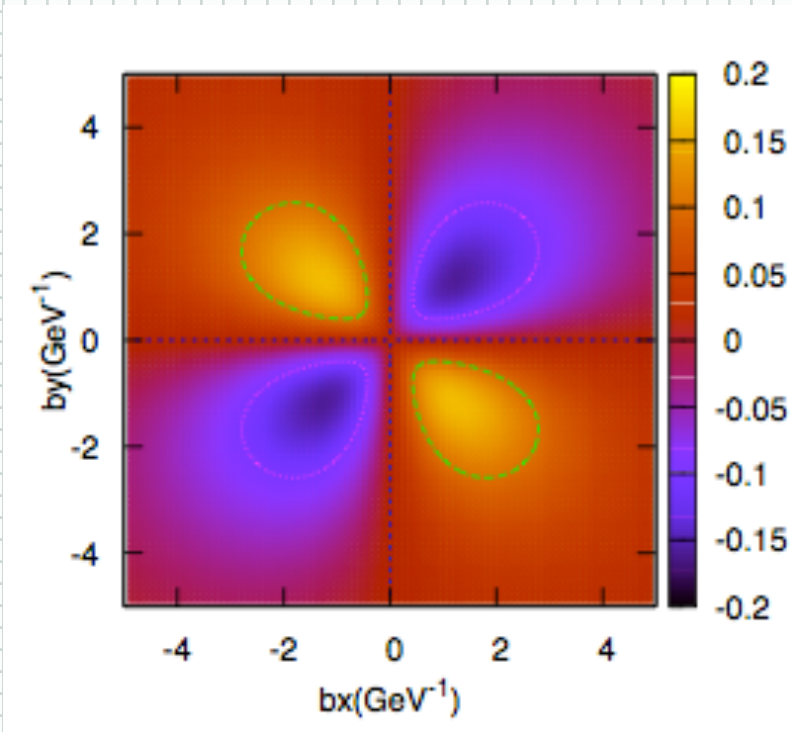
$H_u [1/\text{GeV}^4]$



$H_d [1/\text{GeV}^4]$



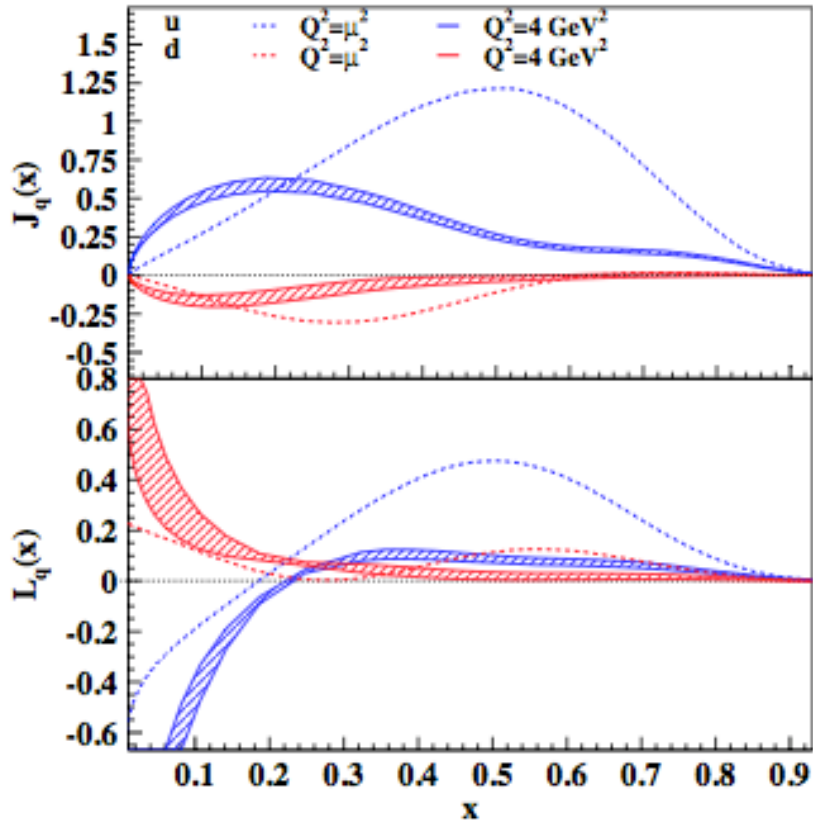
E



ρ_{TU} ?

...finally, a way to explore gluon OAM !!

Orbital Angular Momentum in the Deuteron



New sum rule for spin 1 system \rightarrow deuteron

$$J_q = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)], \quad \longrightarrow \quad J_q = \frac{1}{2} \int dx x H_2^q(x, 0, 0),$$

+ H.O.

\downarrow
 $F_1 + F_2 = G_M$

\downarrow
 G_M

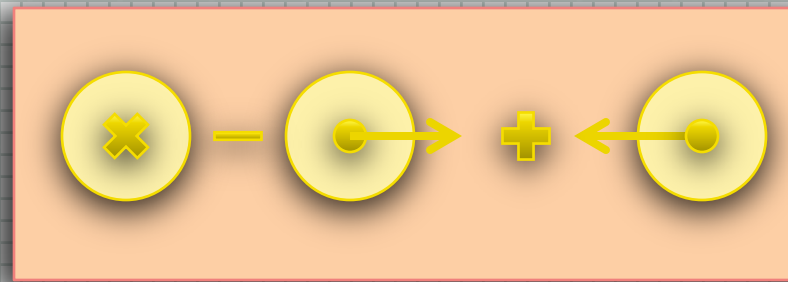
Additional interest in Spin 1 targets: deuterium, ${}^6\text{Li}$, ...

- ✓ In DIS: Unique possibility to study how the deep inelastic structure of nuclei differs from a system of free nucleons.
 - One more distribution w.r.t. spin 1/2

Tensor Structure Function

Hoobhoy, Jaffe, Manohar (1989)

$$b_1(x) = \frac{1}{2} \left[2q_{\uparrow}^0 - (q_{\uparrow}^1 + q_{\downarrow}^1) \right]$$



$b_1(x) \rightarrow 0$ for free nucleons

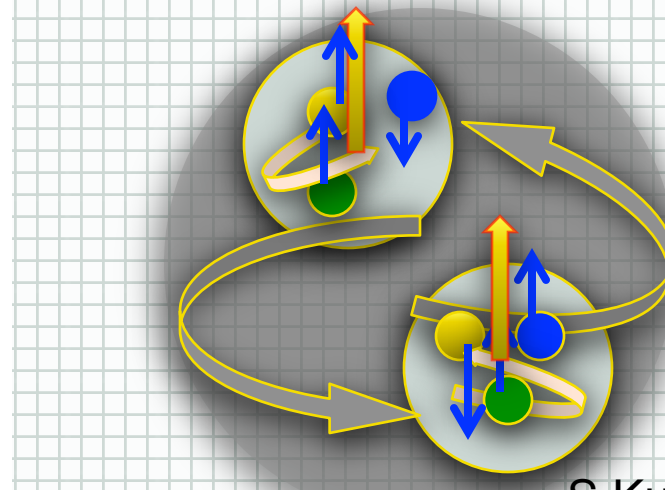
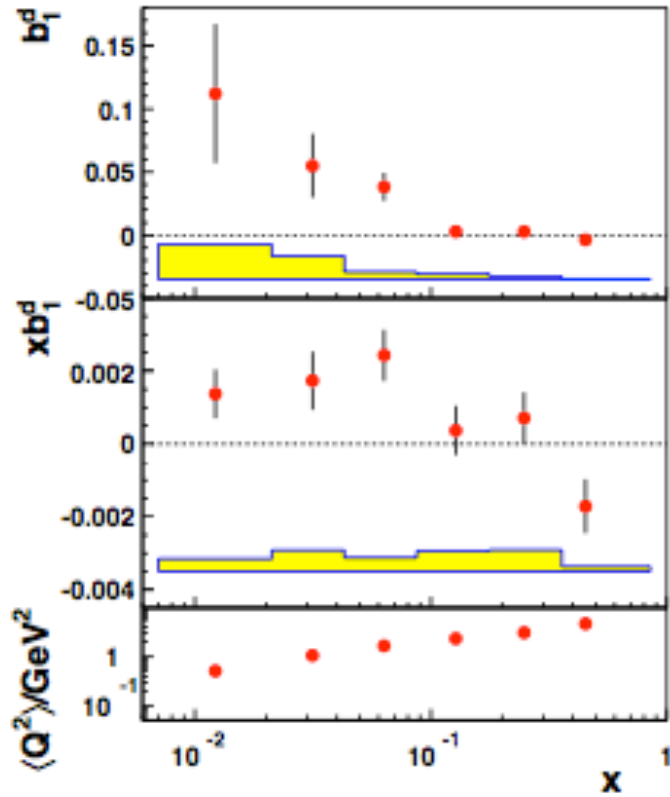
$b_1(x) \neq 0$ in bound systems

→ Role of D wave!

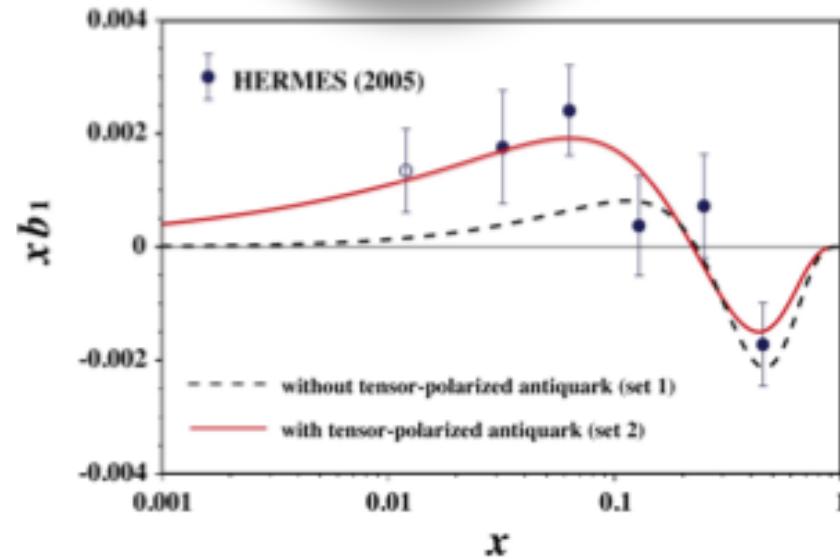
Sum Rule

$$\int b_1(x) dx = \frac{1}{9} \int \left\{ 4 \left[\bar{u}_{\uparrow}^0 - (\bar{u}_{\uparrow}^1 + \bar{u}_{\downarrow}^1) \right] + \left[\bar{d}_{\uparrow}^0 - (\bar{d}_{\uparrow}^1 + \bar{d}_{\downarrow}^1) \right] + \left[\bar{s}_{\uparrow}^0 - (\bar{s}_{\uparrow}^1 + \bar{s}_{\downarrow}^1) \right] \right\} dx$$

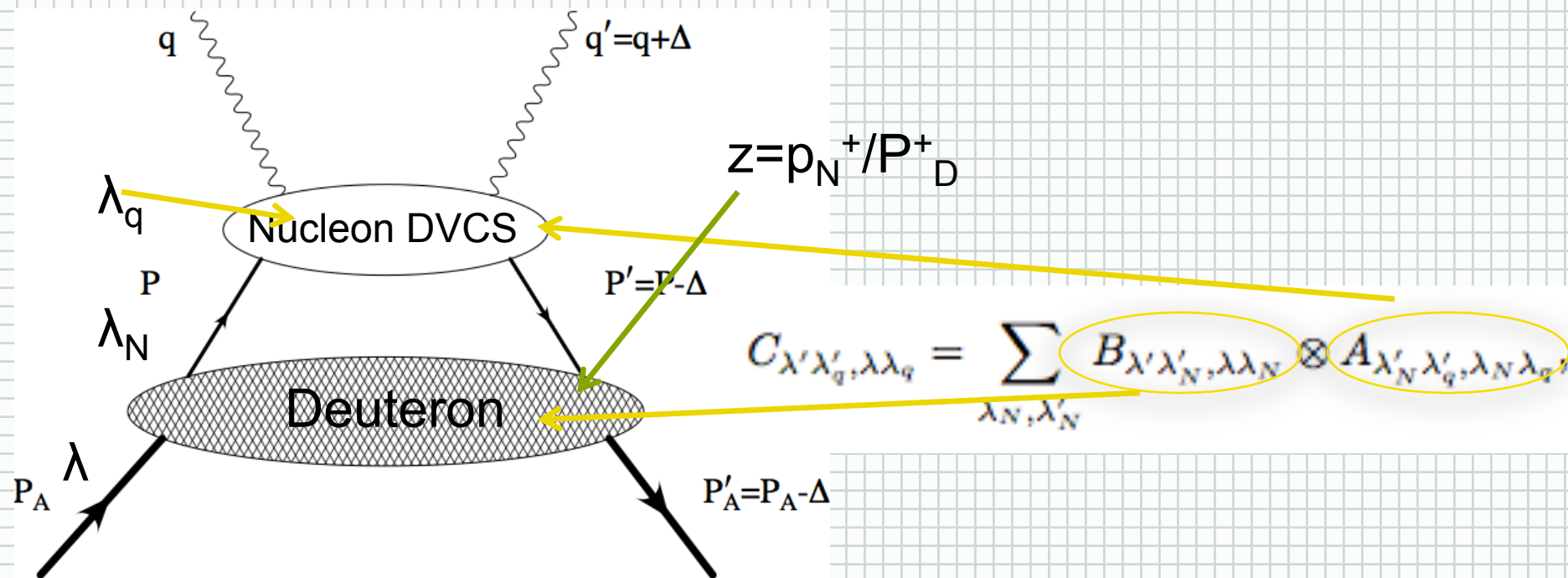
Hermes 2005



S.Kumano PRD82,2010



What are the quark and gluon angular momenta in the deuteron?



$$C_{\lambda' \lambda'_q, \lambda \lambda_q} = \sum_{\lambda_N, \lambda'_N} B_{\lambda' \lambda'_N, \lambda \lambda_N} \otimes A_{\lambda'_N \lambda'_q, \lambda_N \lambda_q}$$

$$\sqrt{\frac{t_0 - t}{2M^2}} H_2(x, 0, 0) = 2 \left[\left(C_{1+,1+} + C_{1-,1-} \right) - \left(C_{1+,0+} + C_{1-,0-} \right) \right]$$

$$\Rightarrow H_2 \approx \int_x^{M_D/M_N} dz \left[f^{++}(z) H_{ISO}(x/z, 0, 0) + f^{0+}(z) E_{ISO}(x/z, 0, 0) \right]$$

$$H_{ISO} = H_u + H_d \quad E_{ISO} = E_u + E_d$$

Deuteron LC Momentum distribution

$$f^{++}(z) = 2\pi M \int_{p_{min}(z)}^{\infty} dp p \sum_{\lambda_N} \chi_+^{*\lambda'_{N1} \lambda_{N2}}(z, p) \chi_+^{\lambda_{N1} \lambda_{N2}}(z, p) \quad (14)$$

$$z = p_N^+ / P_D^+$$

$$f^{0+}(z) = 4\pi M \int_{p_{min}(z)}^{\infty} dp p \sum_{\lambda_N} \chi_0^{*\lambda'_{N1} \lambda_{N2}}(z, p) \chi_+^{\lambda_{N1} \lambda_{N2}}(z, p). \quad (15)$$

$$\lambda_N = \{\lambda_{N1}, \lambda'_{N1}, \lambda_{N2}\}$$

Deuteron w.f. (momentum space)

$$\chi_{\lambda}^{\lambda_{N1}, \lambda_{N2}}(z, p) = \mathcal{N} \sum_{L, m_L, m_S} \begin{pmatrix} j_1 & j_2 & 1 \\ \lambda_{N1} & \lambda_{N2} & m_S \end{pmatrix} \begin{pmatrix} L & S & J \\ m_L & m_S & \lambda \end{pmatrix} \times Y_{L m_L} \left(\frac{\mathbf{p}}{p} = \frac{M(1-z) - E}{p} \right) u_L(p),$$

Mixture of S and D components , L=0,2

If $f^{++}(z) = f^{+0}(z) = \delta(1-z)$ then $H_2 = H + E$

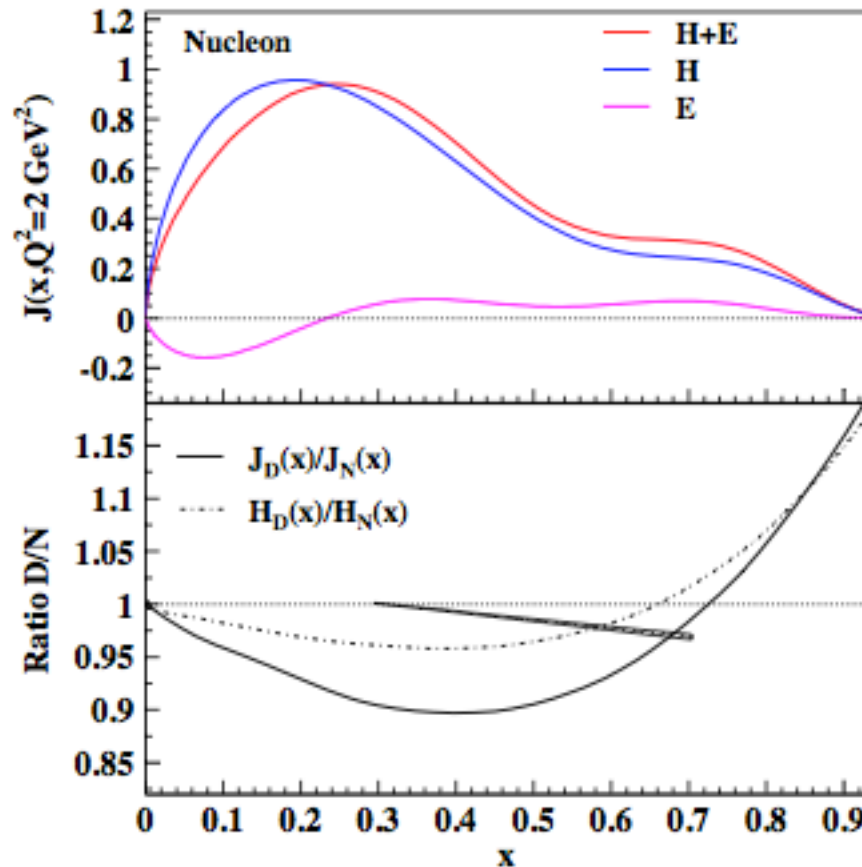
$$J_q = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)], \quad \longrightarrow \quad J_q = \frac{1}{2} \int dx x H_2^q(x, 0, 0),$$

↓

$$F_1 + F_2 = G_M$$

↓

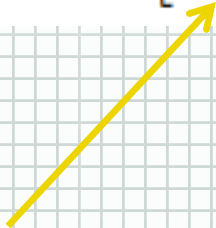
$$G_M$$



Observable sensitive to gluon OAM

$$A_{UT} \approx -\frac{4\sqrt{D_0}}{\Sigma} \Im m \left[\mathcal{H}_1^* \mathcal{H}_5 + \left(\mathcal{H}_1^* + \frac{1}{6} \mathcal{H}_5^* \right) (\mathcal{H}_2 - \mathcal{H}_4) \right]$$

subleading



Conclusions

- GPDs can be extracted from data (once the questions addressed in this talk are considered)
- We have a well tested parametrization that satisfies all constraints - including form factor normalization - for the valence and sea quarks distributions. Work in progress to include gluon component.
- Many open questions on the interpretation of GTMDs and their connection to observables: issue of "observability" of OAM
- Deuteron is an important target: access to gluons
- All of these questions call for new analysis methods (Neural Network and Self Organizing Maps Approaches are the future)
- All of these questions can be addressed in a systematic way at an EIC