GPDs and Their Relationships with TMDs

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Indiana University

These ideas were developed in Trento ECT*, INT, Jlab, DIS2011, Frascati INF, Transversity 2011 & in consultation with many of you

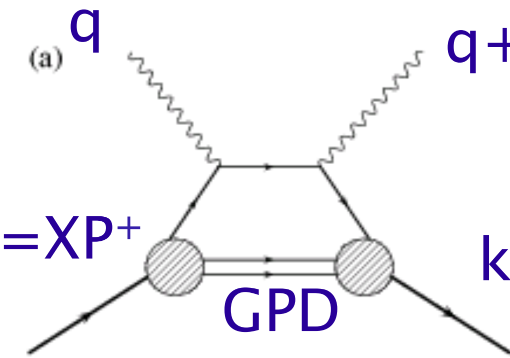
Outline

- Hadron Spin Structure from GPD and TMD perspectives
- "Flexible" parameterization for Chiral Even GPDs
  - Regge ✖ diquark spectator model: R✖Dq
  - Satisfies all constraints
  - Results for DVCS (transverse γ^* → transverse γ) Simonetta's talk
- Extend to Chiral Odd GPDs via diquark spin relations

  Transversity
  - Some relations between Chiral even & odd helicity amps
  - π^0, η, η' production data involve sizable $\gamma^*_{Transverse}$
- Helicity & Transversity Amplitudes, GPDs & TMDs
  - Spin amps <-> Spin bilinears
- Extend R✖Dq to TMDs
  - Trans even & odd
- Wigner Distributions → GTMD → TMDs & GPDs
  - What is measurable?

DVCS & DVMP $\gamma^*(Q^2)+P \rightarrow (\gamma \text{ or meson}) + P'$

partonic picture



$\{ k_T$, $k^+ = XP^+$

$k'_T = k_T - \Delta$, $k'^+ = (X-\zeta)P^+ \}$

P^+

$P'^+ = (1-\zeta)P^+$, $P'_T = -\Delta \}$

$\zeta \rightarrow 0$
Regge

Factorized
"handbag"
picture

Quark–spectator
quark+diquark

$X > \zeta$ DGLAP $\quad \Delta_T \to b_T$ transverse spatial

$X < \zeta$ ERBL $\quad x = (X-\zeta/2)/(1-\zeta/2)$; $x = \zeta/(2-\zeta)$

see Ahmad, GG, Liuti, PRD79, 054014, (2009) for first chiral odd GPD parameterization focused on pseudoscalar production

GPD definitions – 8 quark + 8 gluon

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, \gamma^+ \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda') \left[H^q\, \gamma^+ + E^q\, \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} \right] u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda') \left[\tilde H^q\, \gamma^+\gamma_5 + \tilde E^q\, \frac{\gamma_5\Delta^+}{2m} \right] u(p,\lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx \left[H(x,0,0) + E(x,0,0) \right] x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar\psi(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

$$= \frac{1}{2P^+}\bar u(p',\lambda') \left[H_T^q\, i\sigma^{+i} + \tilde H_T^q\, \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde E_T^q\, \frac{\gamma^+ P^i - P^+\gamma^i}{m} \right] u(p,\lambda).$$

Chiral odd GPDs
-> transversity

4

Helicity amps (q'+N->q+N') are linear combinations of GPDs

$$A_{+,+;+,+} = \sqrt{1-\xi^2}\left[\frac{H+\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E+\tilde{E}}{2}\right]$$

$$A_{-,+;-,+} = \sqrt{1-\xi^2}\left[\frac{H-\tilde{H}}{2} - \frac{\xi^2}{1-\xi^2}\frac{E-\tilde{E}}{2}\right]$$

$$A_{+,+;-,+} = -\frac{\sqrt{t_0-t}}{4M}(E-\xi\tilde{E})$$

$$A_{-,+;+,+} = \frac{\sqrt{t_0-t}}{4M}(E+\xi\tilde{E})$$

for chiral even GPDs and

$$A_{+-,++} = -\frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T + \frac{\xi}{1-\xi^2}\widetilde{E}_T\right]$$

$$A_{+-,-+} = -\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\,\widetilde{H}_T$$

$$A_{++,+-} = \frac{\sqrt{t_0-t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right],$$

for chiral odd GPDs, where for consistency with previous literature we have

T-reversal at $\xi=0$

In diquark spectator models $A_{++;++}$, etc. are calculated directly. Inverted -> GPDs

Spectator inspired model of GPDs

- 2 directions –
  - 1. getting good parameterization of H, E & ~H, ~E

    satisfying many constraints

    (see O. Gonzalez-Hernandez, GG, S. Liuti - Phys.Rev. D84, 034007 (2011))
  - 2. getting 8 spin dependent GPDs
    - Chiral Odd GPDs π^0 production is testing ground (Ahmad, GG, Liuti, PRD79,054014 (2009), Gonzalez, GG, Liuti, arXiv:1201.6088 [hep-ph])
- => **Chiral even related to Chiral odd GPDs → normalizations**

  H, E, . . ← helicity amp relations → H_T, E_T, . .
- Small x & Regge behavior
- Bridge through GPD in helicity or transversity to TMDs?

Invert to obtain model for GPDs

$A_{++,-+} = - A_{++,+-}$
$A_{-+,++} = - A_{+-,++}$
$A_{++,++} = A_{++,--}$

$$H(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} + A_{-,+;-,+}) - \frac{2M\xi^2}{\Delta(1-\xi^2)}(A_{+,+;-,+} - A_{-,+;+,+})$$

$$E(x,\xi,t) = -\frac{2M}{\Delta}(A_{+,+,-,+} - A_{-,+;+,+})$$

$$\tilde{H}(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;+,+} - A_{-,+;-,+}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+,-,+} + A_{-,+;+,+})$$

$$\tilde{E}(x,\xi,t) = \frac{2M}{\Delta\xi}(A_{+,+,-,+} + A_{-,+;+,+})$$

for chiral even GPDs and

$$H_T(x,\xi,t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+,+;-,-} + A_{-,+;+,-}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$\xi E_T(x,\xi,t) - \tilde{E}_T(x,\xi,t) = \frac{2M}{\Delta}(A_{+,+;+,-} - A_{-,+;-,-})$$

$$E_T(x,\xi,t) + \tilde{E}_T(x,\xi,t) = \frac{\Delta}{2M(1-\xi)}[2A_{+,+;+,-} + \frac{4M}{\Delta\sqrt{1-\xi^2}}A_{-,+;+,-}]$$

double flip

$$\tilde{H}_T(x,\xi,t) = \frac{4M^2}{\Delta^2\sqrt{1-\xi^2}}A_{-,+;+,-}$$

Reggeized diquark mass formulation

Where does the Regge behavior come from?

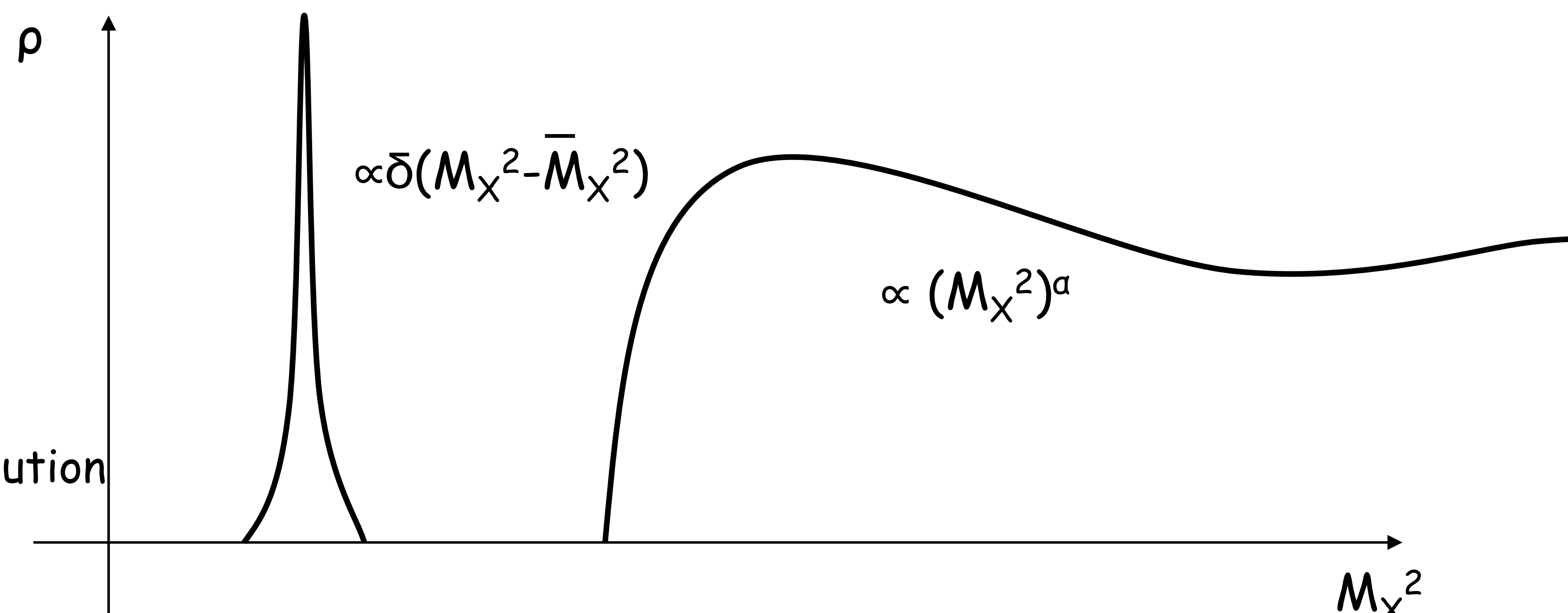
$$G^{\Lambda^2}_{M_X}(X,\zeta,t) = \int d^2\mathbf{k}_\perp \int dM_X^2\, \rho(M_X^2)\, \frac{\phi(k^2,\Lambda^2)}{k^2 - M_X^2}\, \frac{\phi(k'^2,\Lambda^2)}{k'^2 - m^2} \mathcal{N}_{\mathcal{H}(\mathcal{E},\tilde{\mathcal{H}},\tilde{\mathcal{E}})} \qquad \zeta \geq X$$

Diquark spectral function

$$F(X,\zeta,t) = \mathcal{N} G^{M_\Lambda}_{M_X,m}(X,\zeta,t)\, R_p^{\alpha,\alpha'}(X,\zeta,t)$$

"Regge"

+ Q^2 Evolution



Following DIS work by Brodsky, Close, Gunion (1973)

Fitting Procedure e.g. for H and E

- Fit at $\zeta=0$, $t=0$ => $H_q(x,0,0)=q(X)$
  - 3 parameters per quark flavor (M_X^q, Λ_q, α_q) + initial Q_o^2
- Fit at $\zeta=0$, $t\neq 0$ $\Rightarrow$

$$\int_0^1 dX\, H^q(X,t) = F_1^q(t)$$

$$\int_0^1 dX\, E^q(X,t) = F_2^q(t),$$

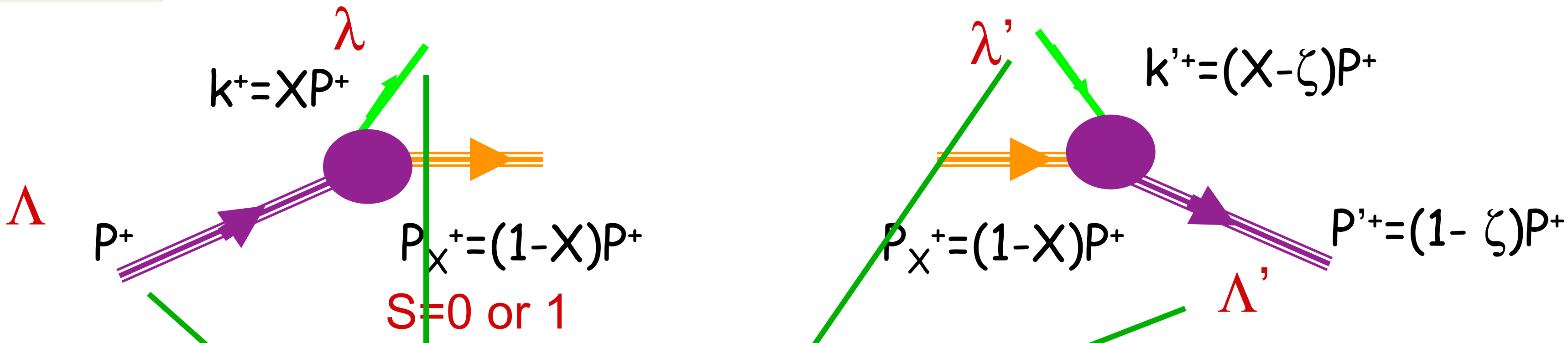
  - 2 parameters per quark flavor (β, p)

$$R = X^{-[\alpha+\alpha'(1-X)^p t+\beta(\varsigma)t]}$$ – Regge factor

$$G^{\lambda}_{M_X}(X,t) = \mathcal{N}\frac{X}{1-X}\int d^2\mathbf{k}_\perp \frac{\phi(k^2,\lambda)}{D(X,\mathbf{k}_\perp)}\frac{\phi(k'^2,\lambda)}{D(X,\mathbf{k}_\perp+(1-X)\mathbf{\Delta}_\perp)}$$ Quark-Diquark

- Fit at $\zeta\neq 0$, $t\neq 0$ $\Rightarrow$ DVCS, DVMP,… data (convolutions of GPDs with Wilson coefficient functions) + lattice results (Mellin Moments of GPDs)
- Note! This is a multivariable analysis $\Rightarrow$ see e.g. Moutarde, Kumericki and D. Mueller, Guidal and Moutarde

Vertex Structures



First focus e.g. on S=0 pure spectator

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

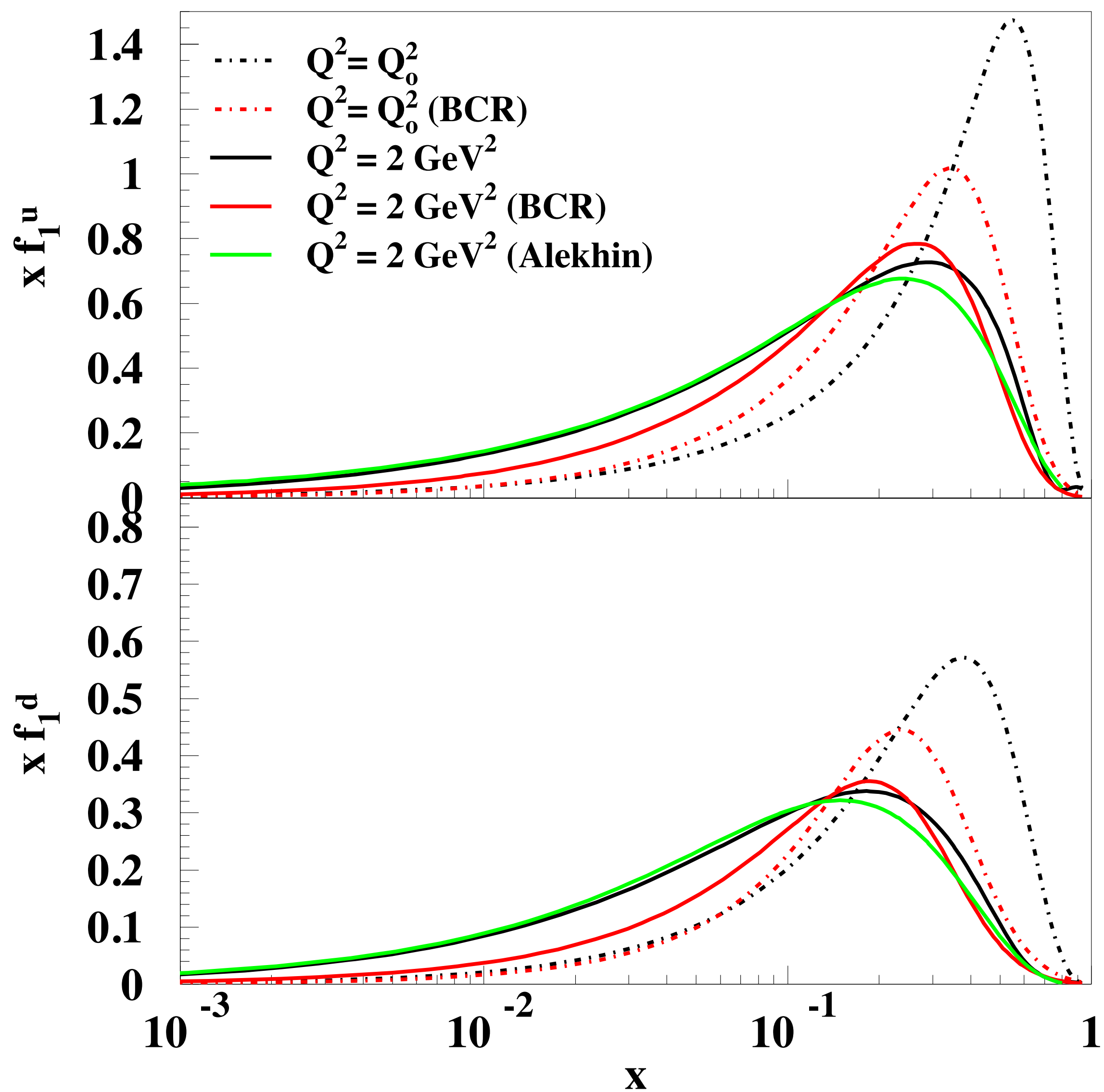
$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Vertex functions

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$



1.4
1.2
1
0.8
0.6
0.4
0.2
0
x f1u
Q2= Qo2
Q2= Qo2 (BCR)
Q2 = 2 GeV2
Q2 = 2 GeV2 (BCR)
Q2 = 2 GeV2 (Alekhin)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
x f1d
10 -3
10 -2
10 -1
1
x

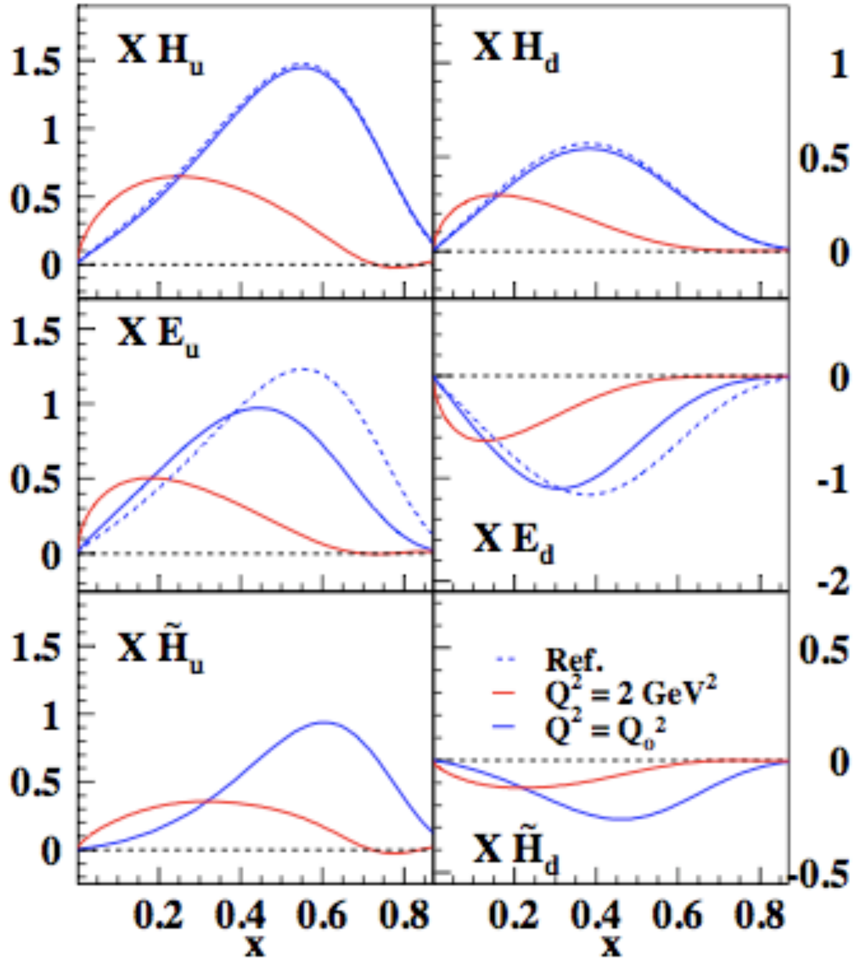


FIG. 6: (color online) GPDs $F_q(X,0,0) \equiv \{H_q, E_q, \tilde{H}_q\}$, for $q = u$ (left) and $q = d$ (right), evaluated at the initial scale, $Q_o^2 = 0.0936$ GeV^2, and at $Q^2 = 2$ GeV^2, respectively. The dashed lines were calculated using the model in Refs.[24, 25] at the initial scale.

Compton Form Factors
→ Real & Imaginary Parts

$$\mathcal{H}_q(\zeta,t,Q^2) = \int_{-1+\zeta}^{+1} dX H_q(X,\zeta,t,Q^2) \times \left(\frac{1}{X-\zeta+i\epsilon} + \frac{1}{X-i\epsilon}\right)$$

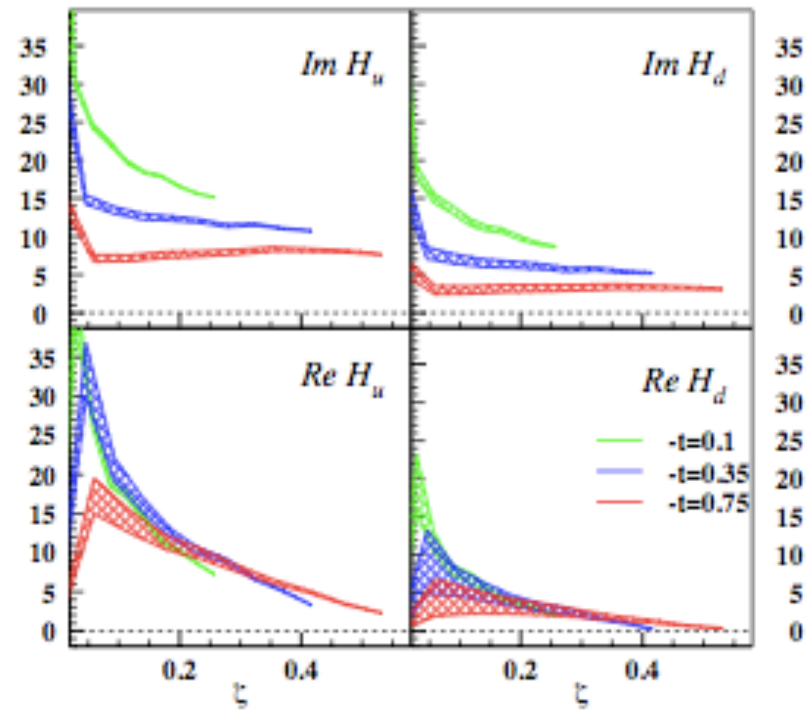


FIG. 9: (color online) Real and imaginary parts of the CFFs, $\mathcal{H}_i(\zeta,t)$, entering Eqs.(85). The CFFs are plotted vs. $x_{Bj} \equiv \zeta$, for different values of t, at $Q^2 = 2$ GeV^2. They are shown with the theoretical uncertainty from the parameters in TableI. Similar results are obtained for E and $\tilde{H}$.

Hermes data

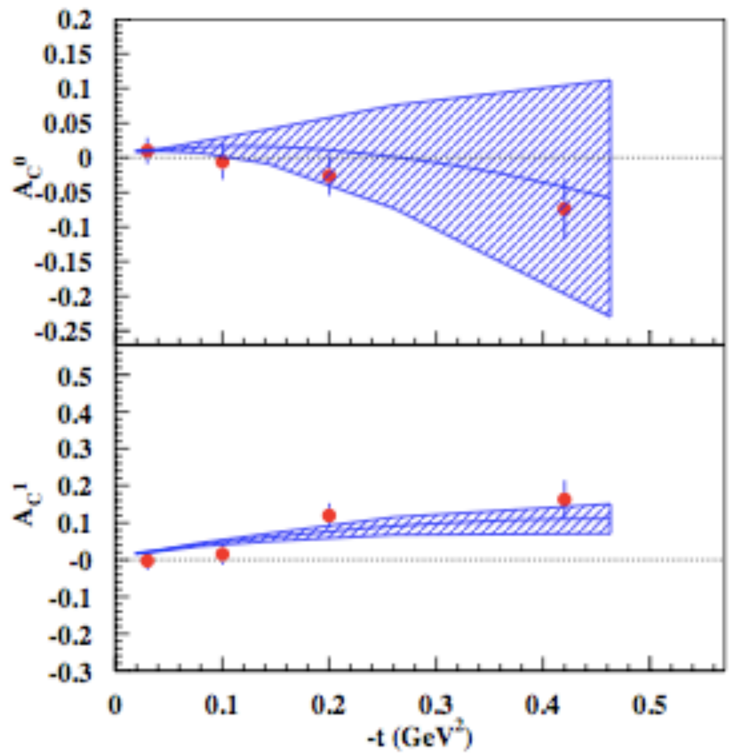


FIG. 20: Coefficients of the beam charge asymmetry, A_C, extracted from experiment [52, 53]. The lower panel is the coefficient for the $\cos\phi$ dependent term in Eq.(82), while the upper panel is the $\cos\phi$ independent term.

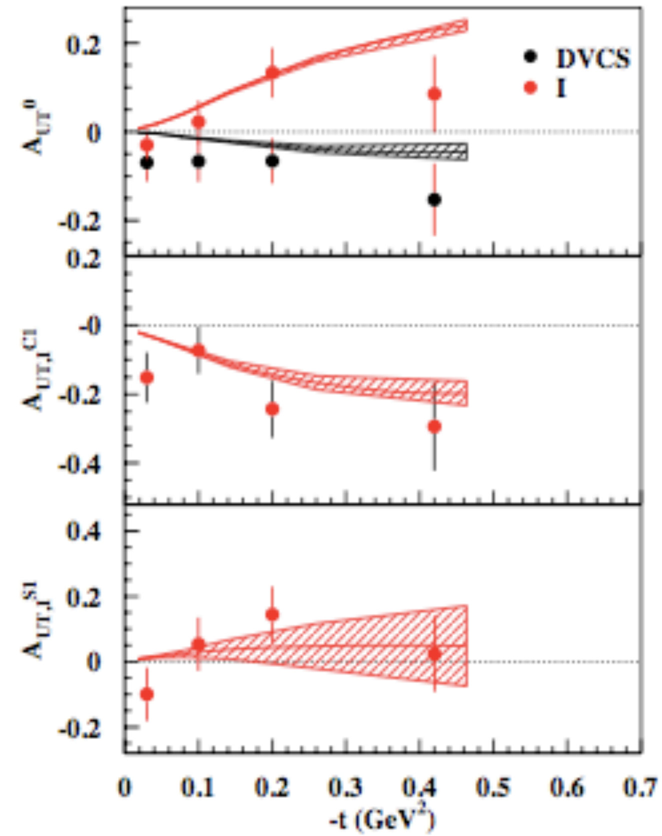
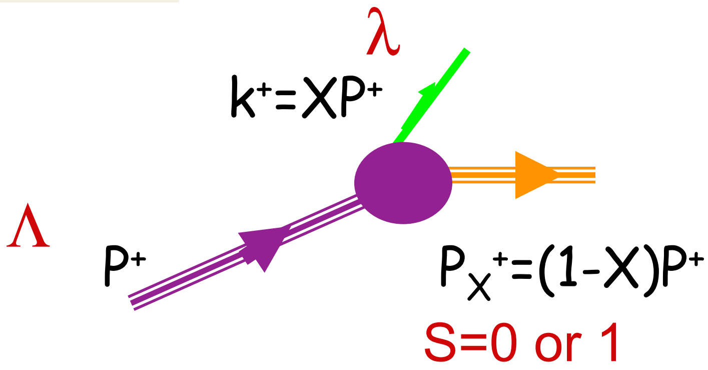
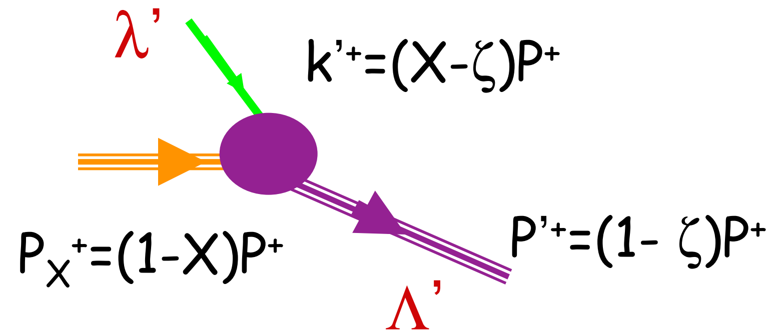


FIG. 21: Coefficients of the A_{UT}, extracted from experiment [52, 53]. The upper panel shows the terms E and F from Eqs.(83) and (84), respectively; the middle panel shows G, and the lower panel H, both in Eq.(84). The curves are predictions obtained extending our quantitative fit of Jefferson lab data to the Hermes set of observables.

Vertex Structures





First focus e.g. on S=0 pure spectator

$$H \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) + \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$E \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) + \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

$$\tilde{H} \Rightarrow \varphi^*_{++}(k',P')\varphi_{++}(k,P) - \varphi^*_{-+}(k',P')\varphi_{-+}(k,P)$$

$$\tilde{E} \Rightarrow \varphi^*_{++}(k',P')\varphi_{+-}(k,P) - \varphi^*_{+-}(k',P')\varphi_{++}(k,P)$$

Note that by switching λ➔- λ & Λ ➔ -Λ (Parity) will have chiral evens go to ± chiral odds giving relations – <u>before k integrations</u>

A(Λ’λ’;Λλ)➔ ±A(Λ’,λ’;-Λ,-λ)

Vertex function

$$\phi(k^2,\lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}.$$

but then (Λ’-λ’)-(Λ-λ) ≠(Λ’-λ’)+(Λ-λ) unless Λ=λ

S=0 Chiral even <-> odd

$$
\begin{aligned}
A^{(0)}_{++,--} &= A^{(0)}_{++,++} \\
A^{(0)}_{++,+-} &= -A^{(0)}_{++,-+} \\
A^{(0)}_{+-,++} &= -A^{(0)}_{-+,++},
\end{aligned}
$$

Invert to get GPDs

$$
\begin{aligned}
\widetilde{H}^0_T &= -(1-\zeta)^2 \frac{M(1-x)}{m+Mx'} \left[E^0 - \frac{\zeta}{2} \widetilde{E}^0 \right] \\
E^0_T &= -\frac{(1-\zeta/2)^2}{1-\zeta} \left[2\widetilde{H}^0_T - E^0 + \left(\frac{\zeta/2}{1-\zeta/2} \right)^2 \widetilde{E}^0 \right] \\
\widetilde{E}^0_T &= -\frac{\zeta/2(1-\zeta/2)}{1-\zeta} \left[2\widetilde{H}^0_T - E^0 + \widetilde{E}^0 \right] \\
H^0_T &= \frac{H^0 + \widetilde{H}^0}{2} - \frac{\zeta^2/4}{1-\zeta} \frac{E^0 + \widetilde{E^0}}{2} - \frac{\zeta^2/4}{(1-\zeta/2)(1-\zeta)} E^0_T + \frac{\zeta/4(1-\zeta/2)}{1-\zeta} \widetilde{E}^0_T + \widetilde{H}^0_T,
\end{aligned}
$$

$S = 0$ double helicity flip amplitude was calculated directly from Eq.(16),

$$
A^{(0)}_{+-,-+} = \frac{t_0 - t}{4M} \frac{1}{\sqrt{1-\zeta}} \frac{1}{(1-\zeta/2)} \frac{\tilde{x}}{m + Mx'} \left[E^0 - \frac{\zeta}{2} \widetilde{E}^0 \right].
$$

S=1 Chiral even <-> odd

$$A^{(1)}_{++,--} = -\frac{x+x'}{1+xx'}\,A^{(1)}_{++,++}$$

$$A^{(1)}_{+-,-+} = 0$$

$$A^{(1)}_{++,+-} = -\sqrt{\frac{\langle \tilde{k}_\perp^2 \rangle}{x'^{\,2} + \langle \tilde{k}_\perp^2 \rangle / P^{+\,2}}}\,A^{(1)}_{++,-+}$$

$$A^{(1)}_{+-,++} = -\sqrt{\frac{\langle k_\perp^2 \rangle}{x^2 + \langle k_\perp^2 \rangle / P^{+\,2}}}\,A^{(1)}_{-+,++},$$

Invert to get GPDs

$$\widetilde{H}_T^{(1)} = 0$$

$$E_T^{(1)} = \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) + a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right]$$

$$\widetilde{E}_T^{(1)} = \frac{1-\zeta/2}{1-\zeta}\left[\tilde{a}\left(E^{(1)} - \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right) - a\left(E^{(1)} + \frac{\zeta/2}{1-\zeta/2}\widetilde{E}^{(1)}\right)\right]$$

$$H_T^{(1)} = -\frac{x+x'}{1+xx'}\left[\frac{H^{(1)}+\widetilde{H}^{(1)}}{2} - \frac{\zeta^2/4}{1-\zeta}\,\frac{E^{(1)}+\widetilde{E}^{(1)}}{2}\right] - \frac{\zeta^2/4}{1-\zeta}E_T^{(1)} + \frac{\zeta/4}{1-\zeta}\widetilde{E}_T^{(1)}$$

Chiral odd integrated amplitudes

$$A_{+-,++} = -A_{-+,--} = \int d^2k_\perp \phi^*_{+-}(k', P')\phi_{++}(k, P)$$

$$= -\frac{\sqrt{t_0 - t}}{2M}\left[\widetilde{H}_T + \frac{1+\xi}{2}E_T - \frac{1+\xi}{2}\widetilde{E}_T\right]$$

$$A_{++,--} = \int d^2k_\perp \phi^*_{++}(k', P')\phi_{--}(k, P)$$

$$= \sqrt{1-\xi^2}\left[H_T + \frac{t_0 - t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T\right]$$

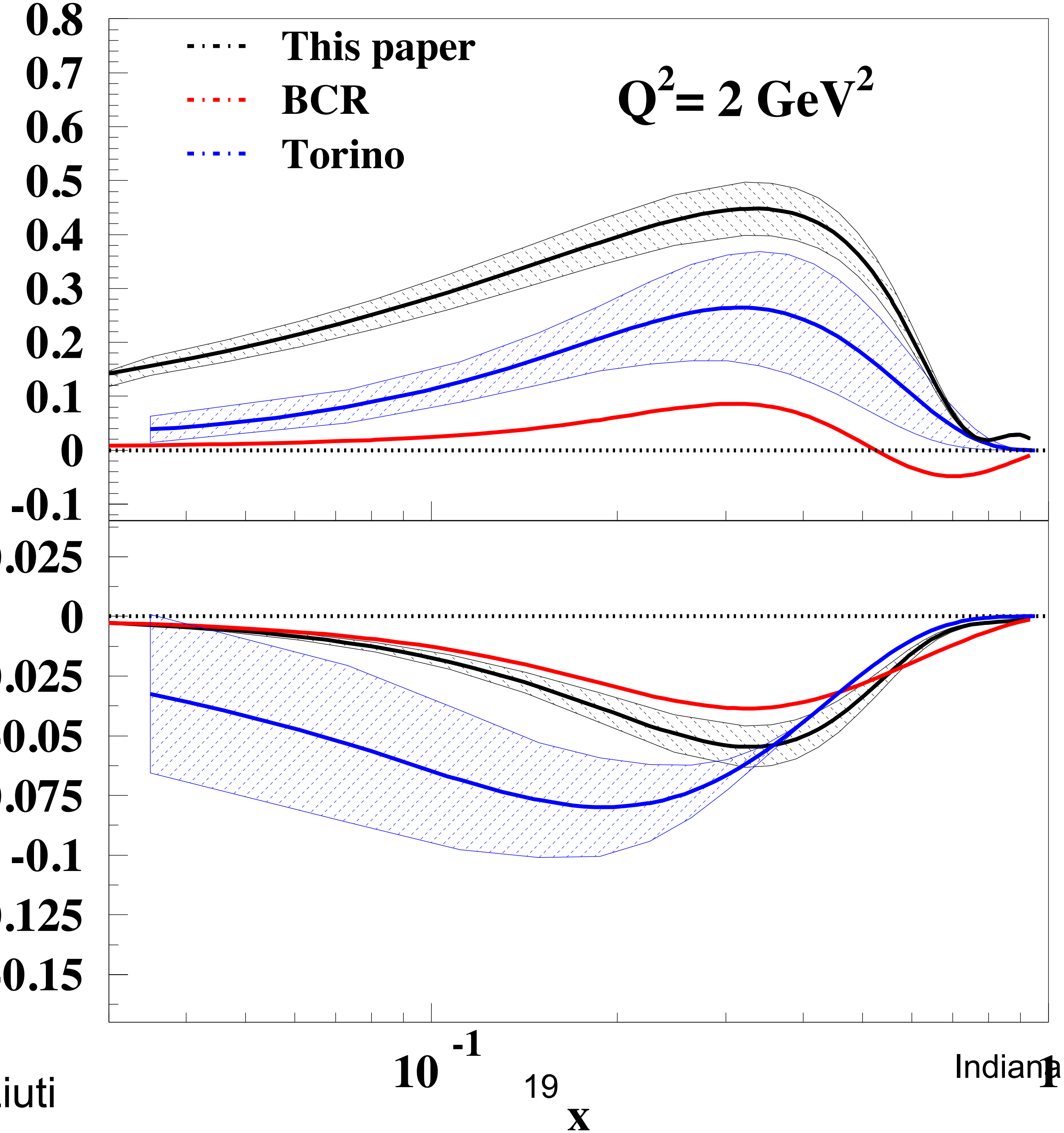
$$A_{+-,-+} = \int d^2k_\perp \phi^*_{+-}(k', P')\phi_{-+}(k, P)$$

$$= -\sqrt{1-\xi^2}\,\frac{t_0 - t}{4M^2}\widetilde{H}_T$$

$$A_{++,+-} = \int d^2k_\perp \phi^*_{++}(k', P')\phi_{+-}(k, P)$$

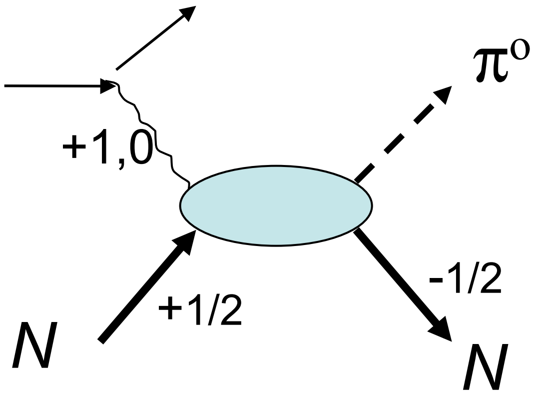
$$= \frac{\sqrt{t_0 - t}}{2M}\left[\widetilde{H}_T + \frac{1-\xi}{2}E_T + \frac{1-\xi}{2}\widetilde{E}_T\right]$$

Extraction of transversity using DVCS data

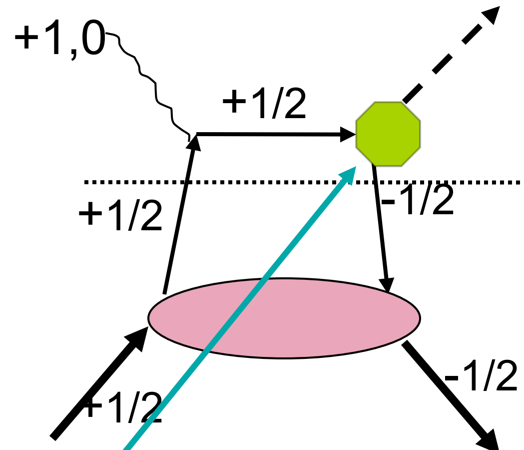


Simonetta Liuti

Exclusive Lepto-production of π^o or η, η' to measure **chiral odd GPDs**

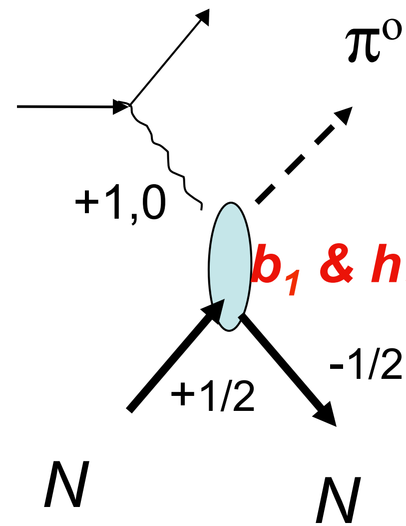


e.g. $f_{+1+,0-}(s,t,Q^2)$



q nos of C-odd
1^{--} exchange
ρ & ω
1^{+-} exchange
b_1 & h_1

$g_{+1+,0-}$ $A_{++,--}$ → H_T



What about coupling of π to $q \rightarrow q'$? Assumed γ^5 vertex
Then for $m_{quark}=0$ has to flip helicity
for $q \rightarrow \pi + q'$ and $\mathbf{q} \times \mathbf{q}' \neq \mathbf{0}$.

Naïve twist 3 $\bar{\psi} \gamma^5 \psi$

Rather than $\gamma^\mu\gamma^5$ – does not flip ***twist 2.*** But $q' \gamma^\mu\gamma^5 q$ will not contribute to transverse γ^*. Differs from t-channel approach to Regge factorization

Which GPDs involved in π^0

- Consider t-channel $\gamma^* \pi^0 \rightarrow$ N+antiN

S/L	0	1	2	3	4	...
0	0^{-+}	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
1	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Axial Vector operators (S;L)

n	$J^{PC}(S;L)$				
0	$0^{-+}(0;0)$	$1^{++}(1;1)$			
1	0^{--}	$1^{+-}(0;1)$	$2^{--}(1;2)$		
2	$0^{-+}(0;0)$	$1^{++}(1;1)$	$2^{-+}(0;2)$	$3^{++}(1;3)$	
3	0^{--}	$1^{+-}(0;1)$	$2^{--}(1;2)$	$3^{+-}(0;3)$	$4^{--}(1;4)$
...	...				

J^{PC} for chiral even GPDs

distribution	J^{PC}	
$H^q(x,\xi,t) - H^q(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	} S=1 crossing even
$E^q(x,\xi,t) - E^q(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	
$\widetilde{H}^q(x,\xi,t) + \widetilde{H}^q(-x,\xi,t)$	$1^{++}, 3^{++}, \ldots$	←S=1 crossing odd
$\widetilde{E}^q(x,\xi,t) + \widetilde{E}^q(-x,\xi,t)$	$0^{-+}, 1^{++}, 2^{-+}, 3^{++}, \ldots$	S=0 crossing even & S=1 crossing odd
$H^q(x,\xi,t) + H^q(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	} S=1 crossing odd
$E^q(x,\xi,t) + E^q(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	
$\widetilde{H}^q(x,\xi,t) - \widetilde{H}^q(-x,\xi,t)$	$2^{--}, 4^{--}, \ldots$	S=1 crossing even
$\widetilde{E}^q(x,\xi,t) - \widetilde{E}^q(-x,\xi,t)$	$1^{+-}, 2^{--}, 3^{+-}, 4^{--}, \ldots$	S=0 crossing odd & S=1 crossing even

Hence only *E~* will enter in p^0 but will be suppressed by Dx or x^2.

Tables: See Lebed & Ji, PRD63,076005 (2001); Diehl & Ivanov, Eur. Phys. Jour. C52, 919 (2007)

J^{PC} for chiral odd GPDs

- 2 series for each GPD, space-space or time-space tensor from $\sigma^{\mu\nu}$. μ-> + in light front frame.
- Indices become (+,1) or (+,2), so mixtures.
- see P. Haegler, PLB 594 (2004) 164–170; Z.Chen & X.Ji, PRD 71, 016003 (2005)

n	σ^{0j} $J^{PC}(S;L,L')$	σ^{jk} $J^{PC}(S;L)$
0	$1^{--}(1;0,2)$	$1^{+-}(0;1)$
1	1^{-+} $2^{++}(1;1,3)$	$1^{++}(1;1)$ $2^{-+}(0;2)$
2	$1^{--}(1;0,2)$ 2^{+-} $3^{--}(1;2,4)$	$1^{+-}(0;1)$ $2^{--}(1;2)$ $3^{+-}(0;3)$
3	1^{-+} $2^{++}(1;1,3)$ 3^{-+} $4^{++}(1;3,5)$	$1^{++}(1;1)$ $2^{-+}(0;2)$ $3^{++}(1;3)$ $4^{-+}(0;4)$
...	...	...

TABLE III: J^{PC} of the tensor operators σ^{0j} and σ^{jk} with $(S;L)$ for the corresponding $N\bar{N}$ state.

	$L=0$	1	2	3	4	...
$\Lambda_\gamma = 0$	1^{+-}		$1,2,3^{+-}$		$3,4,5^{+-}$	
$\lvert\Lambda_\gamma\rvert = 1$	1^{+-}	$0,1,2^{--}$	$1,2,3^{+-}$	$2,3,4^{--}$	$3,4,5^{+-}$	

TABLE IV: J^{PC} of the $\gamma^*\pi^0$ states.

$H_T, E_T, \tilde{H}_T$

$\tilde{E}_T$

lowest J values have lowest L for N-Nbar states & are nearest meson singularities

GPDs & J^{PC}

- Even and odd under crossing

Chiral Even GPD	J^{PC}	
$H(x,\xi,t) - H(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	$(S=1)$
$E(x,\xi,t) - E(-x,\xi,t)$	$0^{++}, 2^{++}, \ldots$	$(S=1)$
$\widetilde{H}(x,\xi,t) + \widetilde{H}(-x,\xi,t)$	$1^{++}, 3^{++}, \ldots$	$(S=1)$
$\widetilde{E}(x,\xi,t) + \widetilde{E}(-x,\xi,t)$	$0^{-+}, 1^{++}, 2^{-+}, 3^{++}, \ldots$	$(S=0,1)$
$H(x,\xi,t) + H(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	$(S=1)$
$E(x,\xi,t) + E(-x,\xi,t)$	$1^{--}, 3^{--}, \ldots$	$(S=1)$
$\widetilde{H}(x,\xi,t) - \widetilde{H}(-x,\xi,t)$	$2^{--}, 4^{--}, \ldots$	$(S=1)$
$\widetilde{E}(x,\xi,t) - \widetilde{E}(-x,\xi,t)$	$1^{+-}, 2^{--}, 3^{+-}, 4^{--}, \ldots$	$(S=0,1)$

Chiral Odd GPD	J^{-C}		J^{+C}	
$H_T(x,\xi,t) - H_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$1^{++}, 3^{++} \ldots$	$(S=1)$
$E_T(x,\xi,t) - E_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$1^{++}, 3^{++} \ldots$	$(S=1)$
$\widetilde{H}_T(x,\xi,t) - \widetilde{H}_T(-x,\xi,t)$			$1^{++}, 3^{++}, \ldots$	$(S=1)$
$\widetilde{E}_T(x,\xi,t) - \widetilde{E}_T(-x,\xi,t)$	$2^{-+}, 4^{-+}, \ldots$	$(S=0)$	$3^{++}, 5^{++} \ldots$	$(S=1)$
$H_T(x,\xi,t) + H_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$	$1^{+-}, 3^{+-} \ldots$	(S=0)
$E_T(x,\xi,t) + E_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$	$1^{+-}, 3^{+-} \ldots$	(S=0)
$\widetilde{H}_T(x,\xi,t) + \widetilde{H}_T(-x,\xi,t)$	$1^{--}, 2^{--}, 3^{--} \ldots$	$(S=1)$		
$\widetilde{E}_T(x,\xi,t) + \widetilde{E}_T(-x,\xi,t)$	$2^{--}, 3^{--}, 4^{--} \ldots$	$(S=1)$	$3^{+-}, 5^{+-} \ldots$	(S=0)

Chiral even ⇔ odd relations

- Helicity amps from $\varphi^*_{q'N'} \times \varphi_{qN}$
  - » With $\varphi_{-q\,-N} = \pm\varphi^*_{qN}$

$S = 0$

$$\begin{aligned}
\phi_{++}(k,P) &= \mathcal{A}\,(m + Mx) \\
\phi_{+-}(k,P) &= \mathcal{A}(k_1 - ik_2)
\end{aligned}$$

$S = 1$

$$\begin{aligned}
\phi^{+}_{++}(k,P) &= \mathcal{A}\,\frac{k_1 - ik_2}{1-x} \\
\phi^{-}_{++}(k,P) &= -\mathcal{A}\,\frac{(k_1 + ik_2)X}{1-x} \\
\phi^{+}_{+-}(k,P) &= 0 \\
\phi^{-}_{+-}(k,P) &= -\mathcal{A}\,(m + Mx) \\
\phi^{+}_{-+}(k,P) &= -\mathcal{A}\,(m + Mx) \\
\phi^{-}_{-+}(k,P) &= 0
\end{aligned}$$

$$\mathcal{A} = \frac{1}{\sqrt{x}}\,\frac{\Gamma(k)}{k^2 - m^2}.$$

$$\begin{aligned}
A^{(0)}_{++,--} &= A^{(0)}_{++,++} \\
A^{(0)}_{++,+-} &= -A^{(0)}_{++,-+} \\
A^{(0)}_{+-,++} &= -A^{(0)}_{-+,++},
\end{aligned}$$

$$\begin{aligned}
A^{(1)}_{++,--} &= -\frac{x + x'}{1 + xx'}\,A^{(1)}_{++,++} \\
A^{(1)}_{+-,-+} &= 0 \\
A^{(1)}_{++,+-} &= -\sqrt{\frac{\langle \tilde{k}_\perp^2 \rangle}{x'^2 + \langle \tilde{k}_\perp^2 \rangle / P^{+2}}}\,A^{(1)}_{++,-+} \\
A^{(1)}_{+-,++} &= -\sqrt{\frac{\langle k_\perp^2 \rangle}{x^2 + \langle k_\perp^2 \rangle / P^{+2}}}\,A^{(1)}_{-+,++},
\end{aligned}$$

$$\mathcal{H}_T, \mathcal{E}_T, \tilde{\mathcal{E}}_T, \bar{\mathcal{E}}_T$$

$$\frac{d\sigma_T}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\left[|\mathcal{H}_T|^2 + \tau\left(|\bar{\mathcal{E}}_T|^2 + |\tilde{\mathcal{E}}_T|^2\right)\right] \quad (1$$

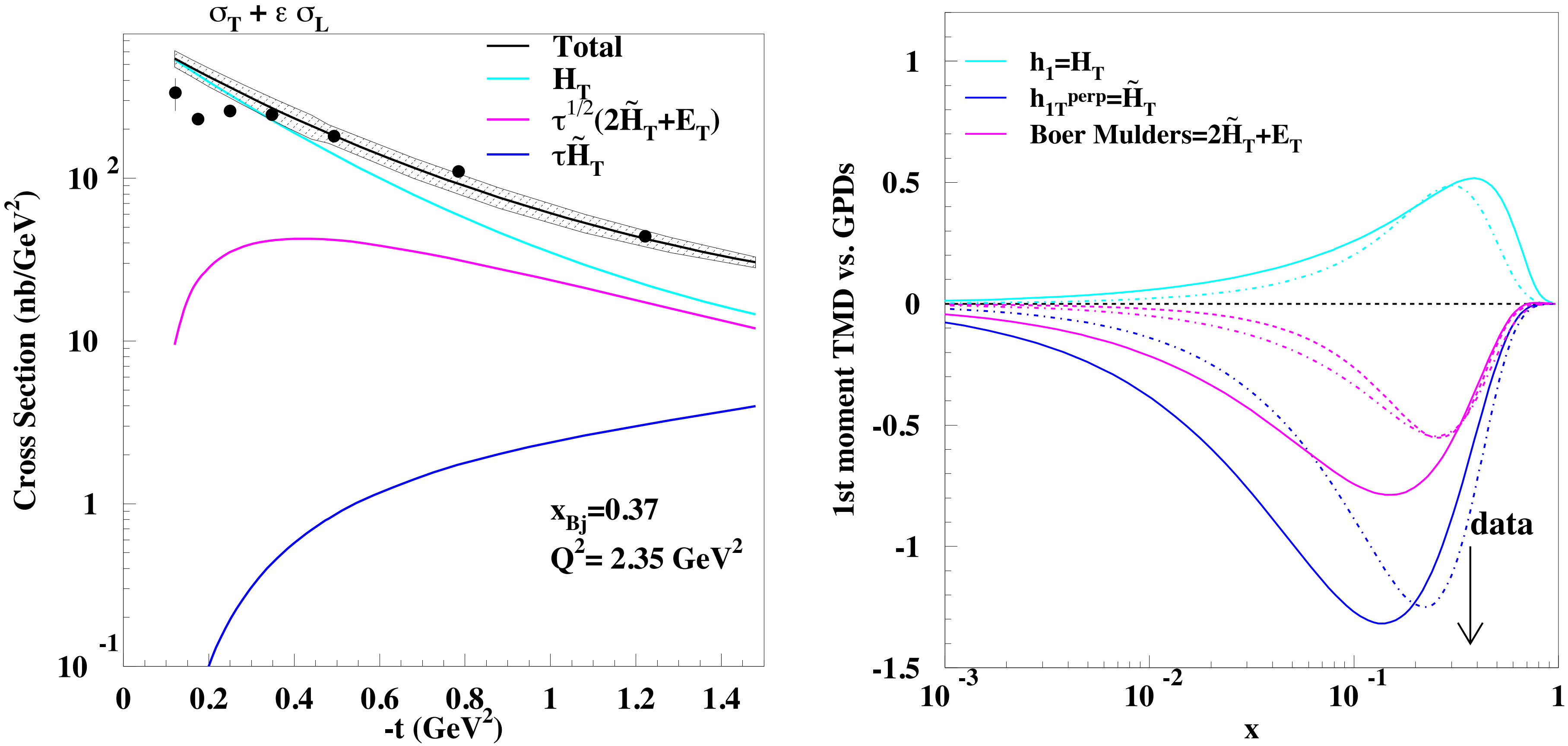
$$\frac{d\sigma_L}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\frac{2M^2\tau}{Q^2}|\mathcal{H}_T|^2 \quad (1$$

$$\frac{d\sigma_{TT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\tau\left[|\bar{\mathcal{E}}_T|^2 - |\tilde{\mathcal{E}}_T|^2 + \Re e\mathcal{H}_T\frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} + \Im m\mathcal{H}_T\frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2}\right. \quad (1$$

$$\frac{d\sigma_{LT}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\, 2\sqrt{\frac{2M^2\tau}{Q^2}}|\mathcal{H}_T|^2 \quad (1$$

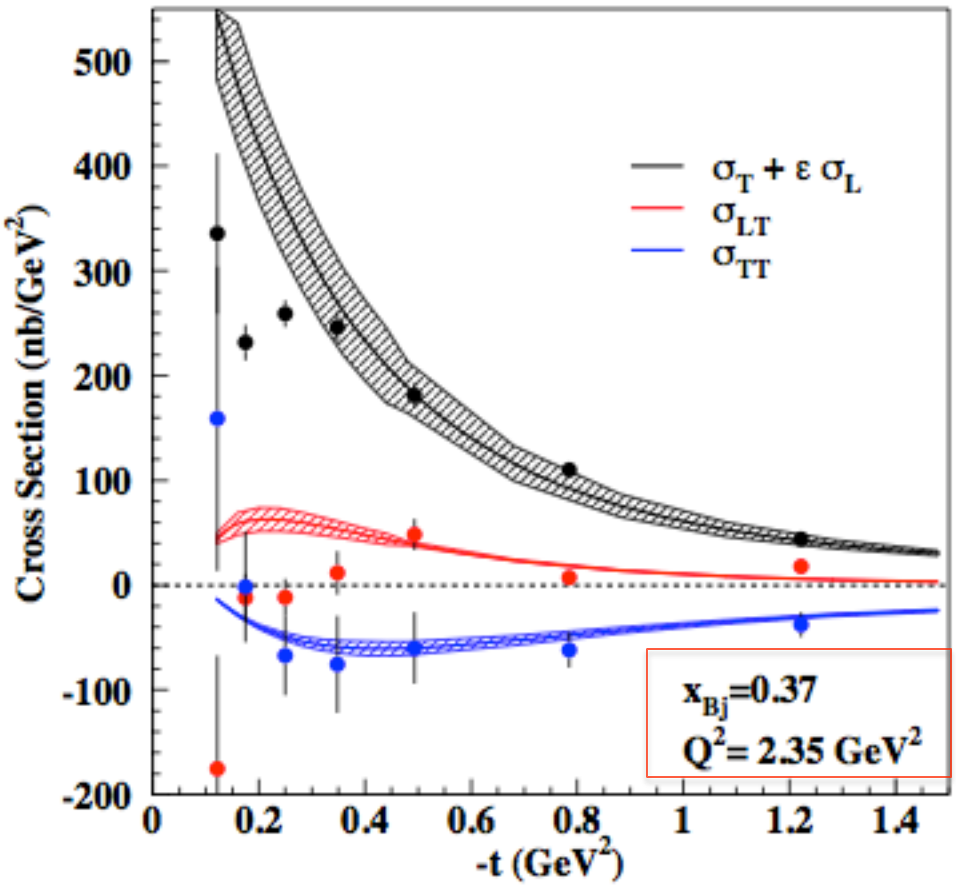
$$\frac{d\sigma_{L'T}}{dt} \approx \mathcal{N}^2[g_\pi^{odd}(Q)]^2 \frac{1}{(1+\xi)^4}\tau\sqrt{\frac{2M^2\tau}{Q^2}}\left[\Re e\mathcal{H}_T\frac{\Im m(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2} - \Im m\mathcal{H}_T\frac{\Re e(\bar{\mathcal{E}}_T - \mathcal{E}_T)}{2}\right] \quad (1$$

$\tau = (t_0 - t)/2M^2$



Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010

How well do the parameters fixed with DVCS data reproduce π^o electroproduction data?



Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010

CLAS π^0 arXiv:1206.6355v1 [hep-ex]

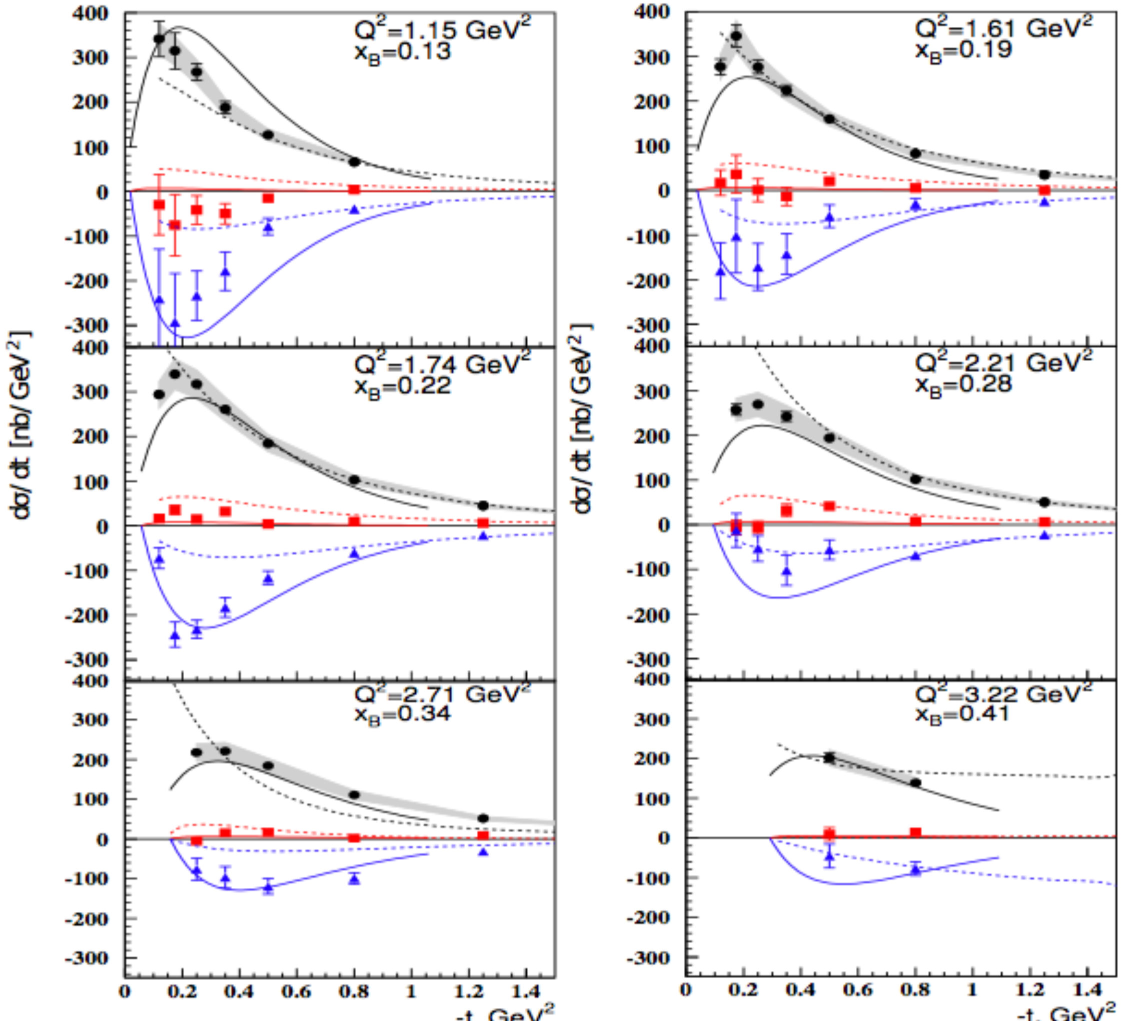


FIG. 2: The extracted structure functions vs. t for the bins with the best kinematic coverage and for which there are theoretical calculations. The data and curves are as follows: black-$\sigma_U (= \sigma_T + \epsilon\sigma_L)$, blue-$\sigma_{TT}$, and red-σ_{LT}. The shaded bands reflect the experimental systematic uncertainties.The curves are theoretical predictions produced with the models of Refs. [14] (solid) and [15] (dashed).

Transversity amplitudes & GPDs

- $H_T(x,0,0) = h_1(x)$ "measures" transfer of transversity
- $| p,+(-)>^{Ty}=[|p,+> +(-)i\, |p,->]/\sqrt{2}$ (y-normal to scattering plane)
- Or $| p,+(-)>^{Tx}=[|p,+> +(-)|p,->]/\sqrt{2}$ (x-in plane)
- Or $| p,+(-)>^{Tx}=[|p,+> +(-)e^{i\phi}\, |p,->]/\sqrt{2}$ (in transverse plane)
- $A^{Ty}_{N',q';N,q}$ = linear combination of $A^{helicity}$
- $H_T \propto A^{Ty}_{++,++} - A^{Ty}_{+-,+-} - A^{Ty}_{-+,-+} + A^{Ty}_{--,--}$ not for $\Delta_T \neq 0$
- $H_T \propto A^{Tx}_{++,++} - A^{Tx}_{+-,+-} - A^{Tx}_{-+,-+} + A^{Tx}_{--,--}$

Introduce $H'_T = H_T + \dfrac{\Delta_T^2}{2M^2}\tilde{H}_T$

$$\propto A^{Ty}_{++,++} - A^{Ty}_{+-,+-} - A^{Ty}_{-+,-+} + A^{Ty}_{--,--}$$

For $\Delta_T = 0$ $\quad T_Y \leftrightarrow T_X$

So $\dfrac{\Delta_T^2}{2M^2}\tilde{H}_T$ is the difference between canonical transversity transfer and planar transversity transfer

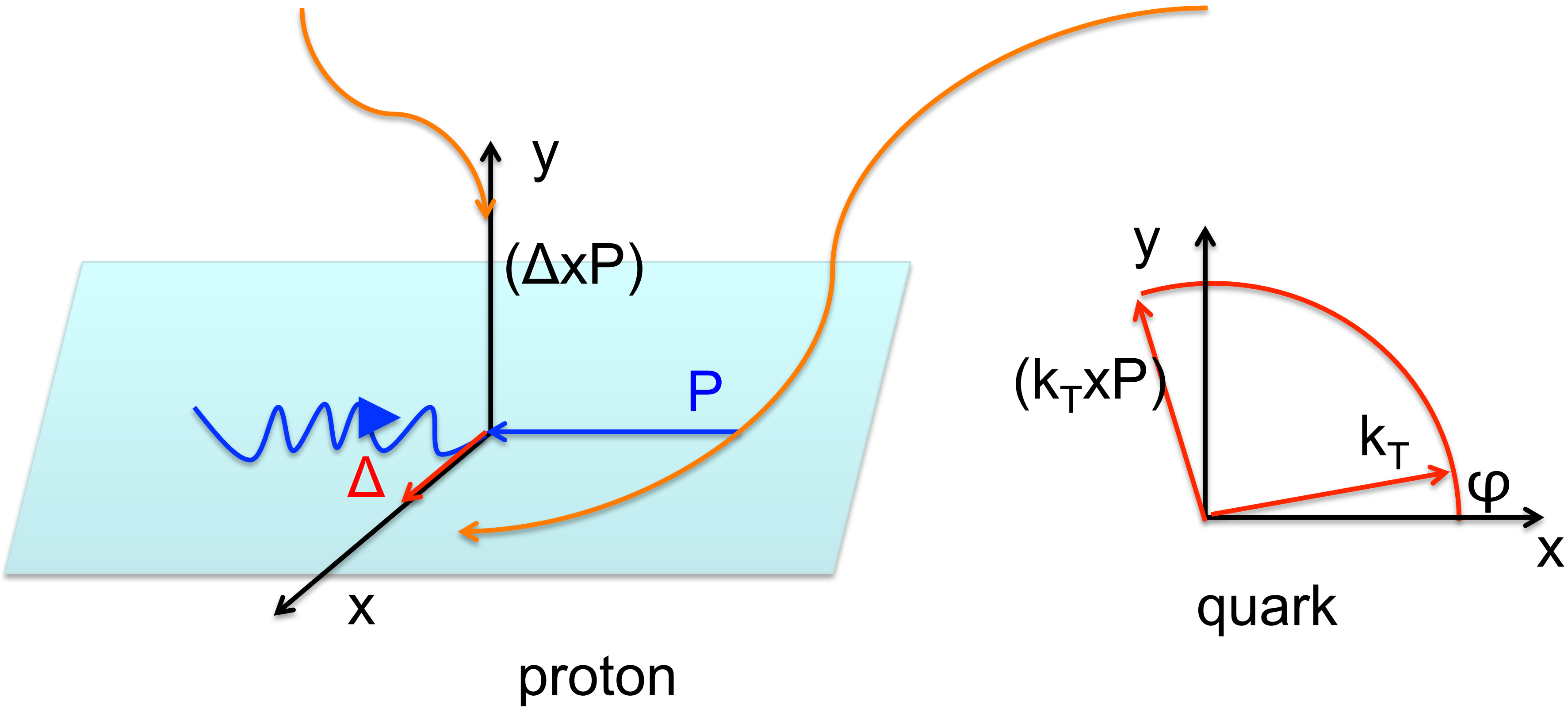
Transversity bases

Canonical

$$| P, +(-)\rangle^{T_Y} = \frac{1}{\sqrt{2}} [| P, +\rangle + (-) i \, | P, -\rangle$$

Planar

$$| P, +(-)\rangle^{T_X} = \frac{1}{\sqrt{2}} [| P, +\rangle + (-) \, | P, -\rangle]$$



Spin amplitudes, GPDs → TMDs

- $\mathcal{F}_T$'s are Diagonal in transversities ⇒ probabilistic interpretations

  w/o b-space

- H_T & H_T' Same *spin form* as TMDs $h_{1T}(x,k_T^2)$ combined with $h_{1T}^{\perp}(x,k_T^2)$

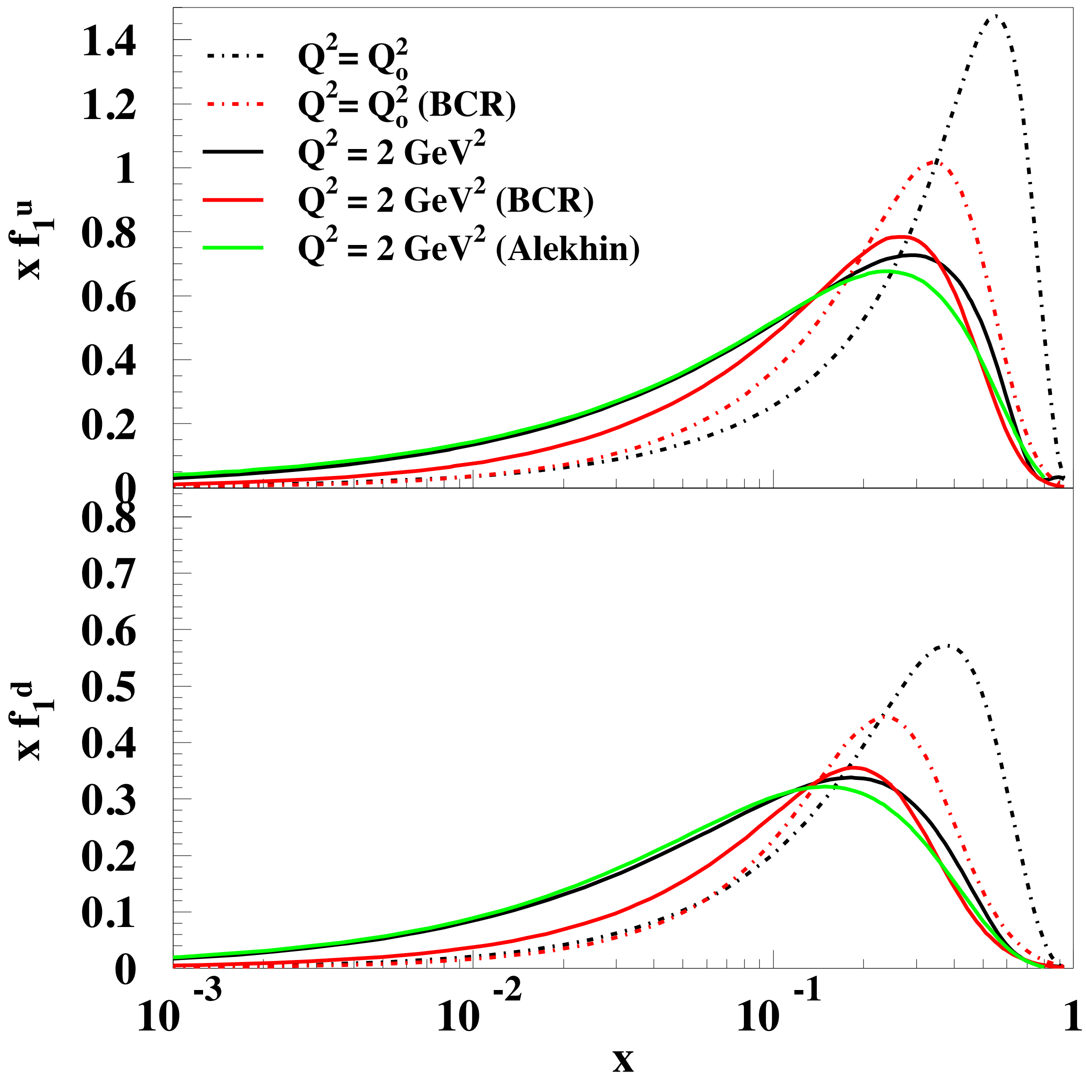
  $h_{1T}(x,k_T^2)$ compare $H_T(x,0,\Delta_T^2)$

  or unintegrated $H_T(x,0,\Delta_T^2,k)$

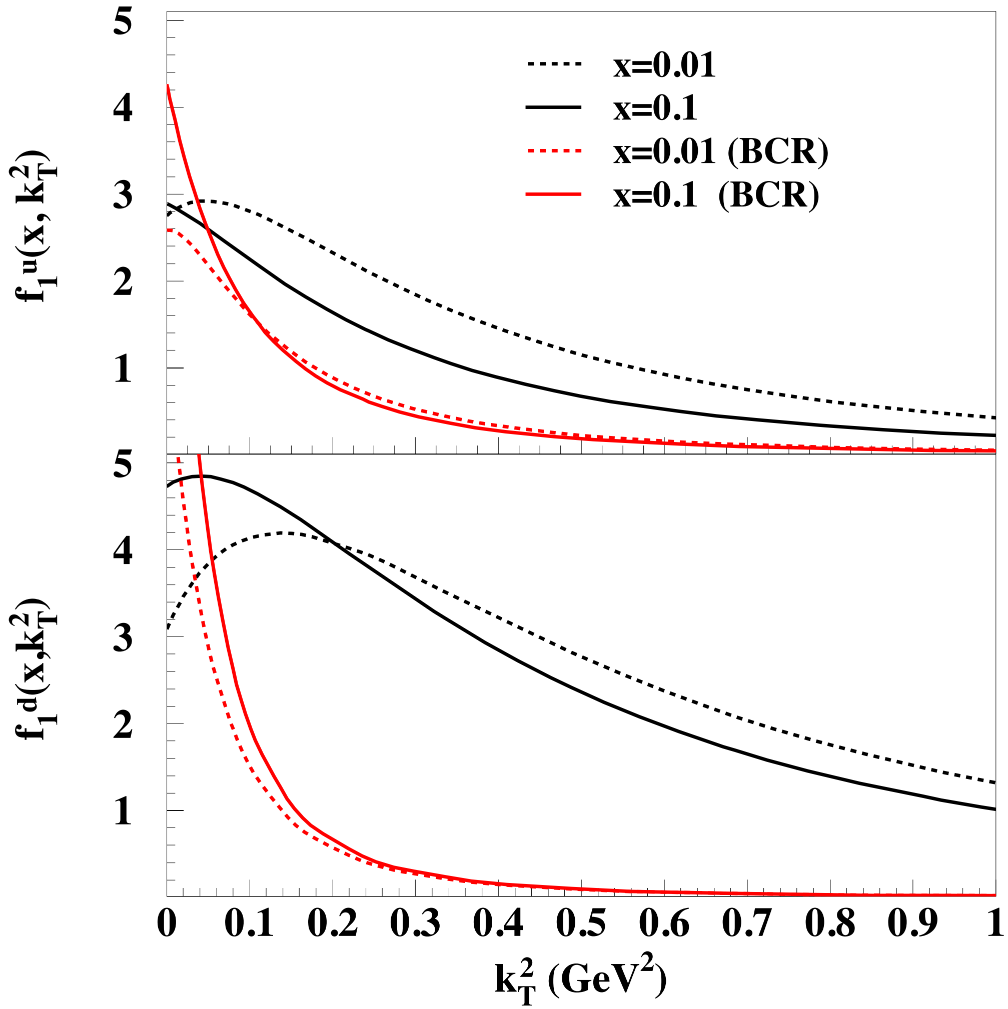
$$h_1(x,\vec{k}_T^2) = h_{1T}(x,\vec{k}_T^2) + \frac{\vec{k}_T^2}{2M^2} h_{1T}^{\perp}(x,\vec{k}_T^2)$$

$$f_{1T}^{\perp(1)}(x) = \int d^2\vec{k}_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp}(x,\vec{k}_T^2) = -\frac{g}{2M} T(x,S_T)$$

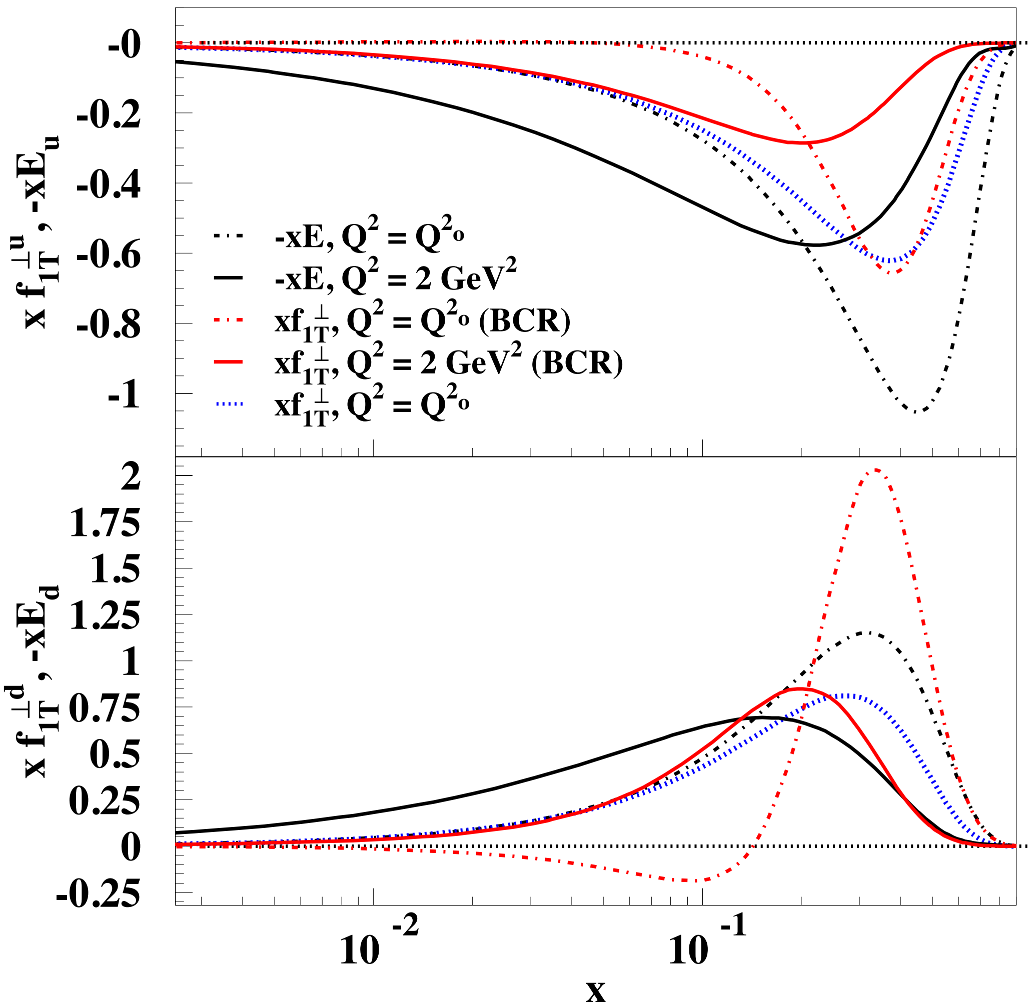
Recall R×Dq determination of pdf



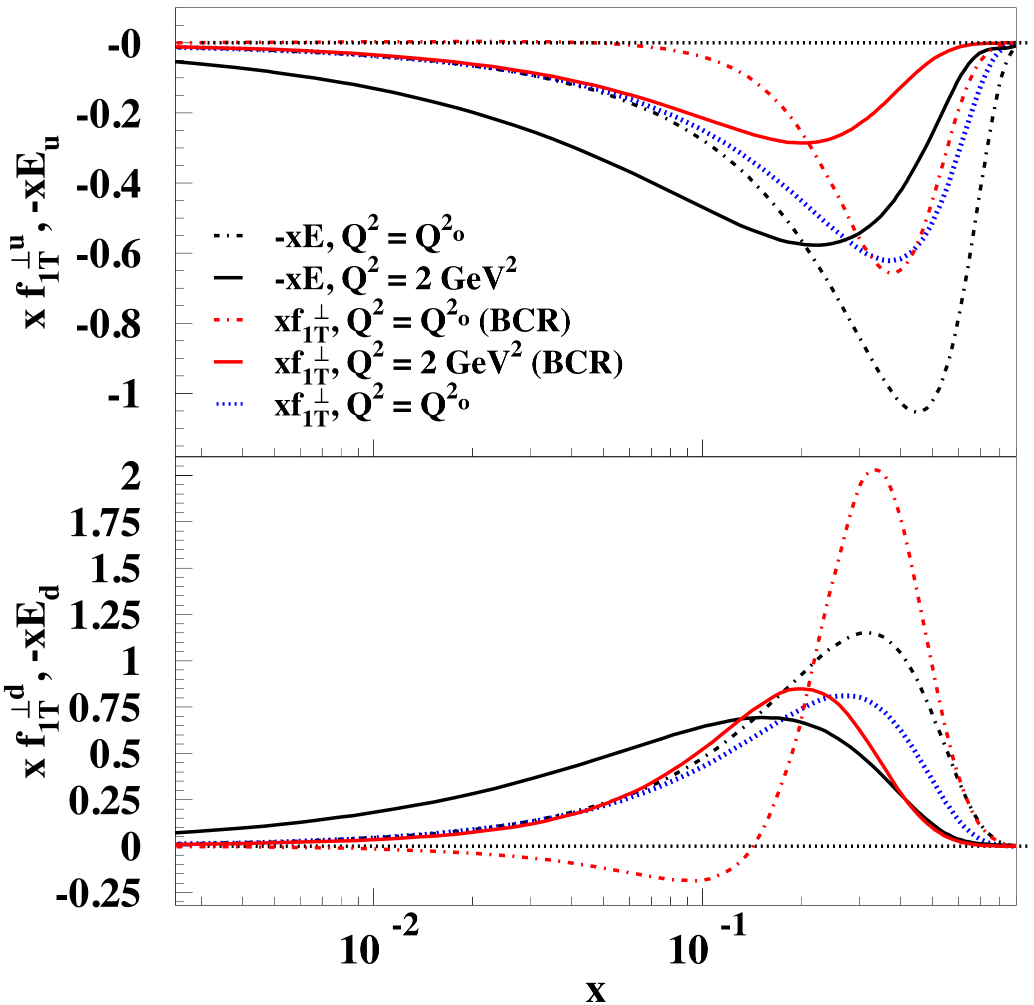
Extend R✖Dq to unintegrated k_T



Beyond R✖Dq to get trans-odd $f_{1T}^{\perp}$ need f.s.i./gauge link
Same spin structure as E for this model (0th moment)



Beyond R✖Dq to get trans-odd $f_{1T}^{\perp}$ need f.s.i./gauge link Same spin structure as E for this model



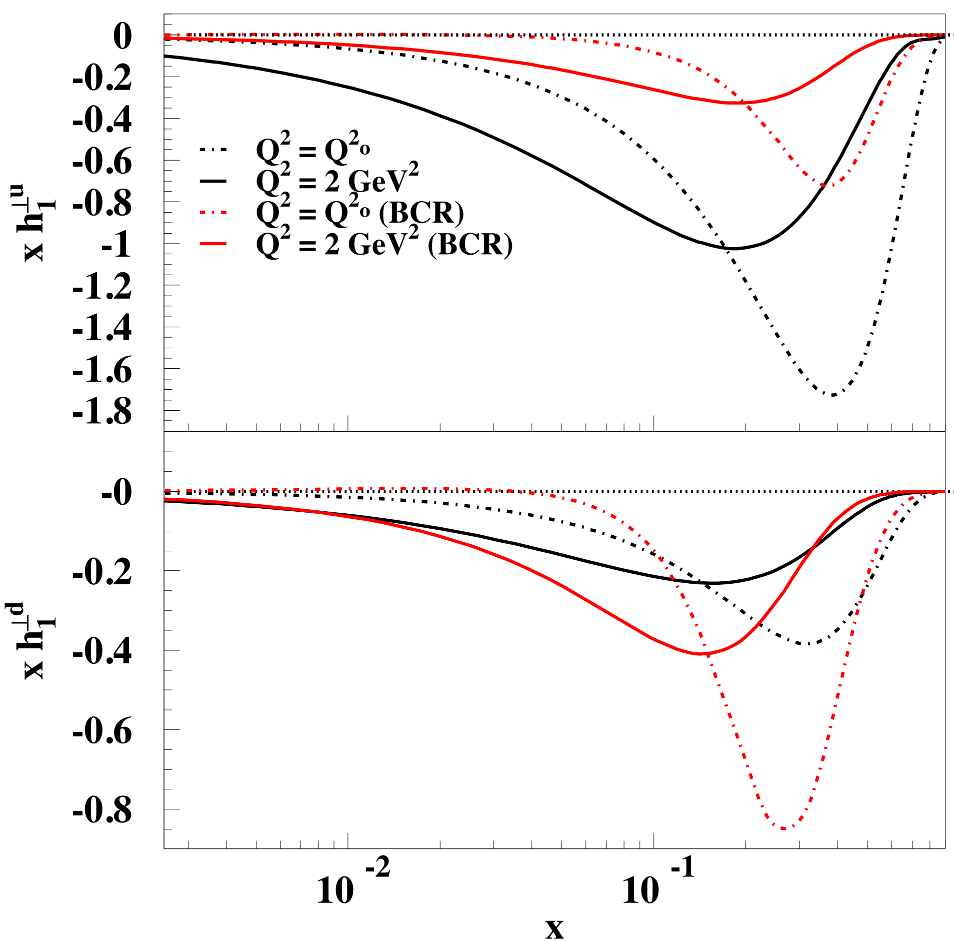
All GPDs' spin structures have corresponding TMDs

Is this more general than R✖Dq or simpler quark models?

Wigner distributions or Generalized TMDs?

$$\sum_{\Lambda} \Im m\, F_{\Lambda+,\Lambda-} \propto h_1^{\perp}(x,k_T^2)$$
$$\sum_{\lambda} \Im m\, F_{+\lambda,-\lambda} \propto f_{1T}^{\perp}(x,k_T^2)$$

$$A_{++,+-} - A_{+-,++} \propto 2\tilde{H}_T + E_T$$
$$A_{++,-+} - A_{-+,++} \propto E$$



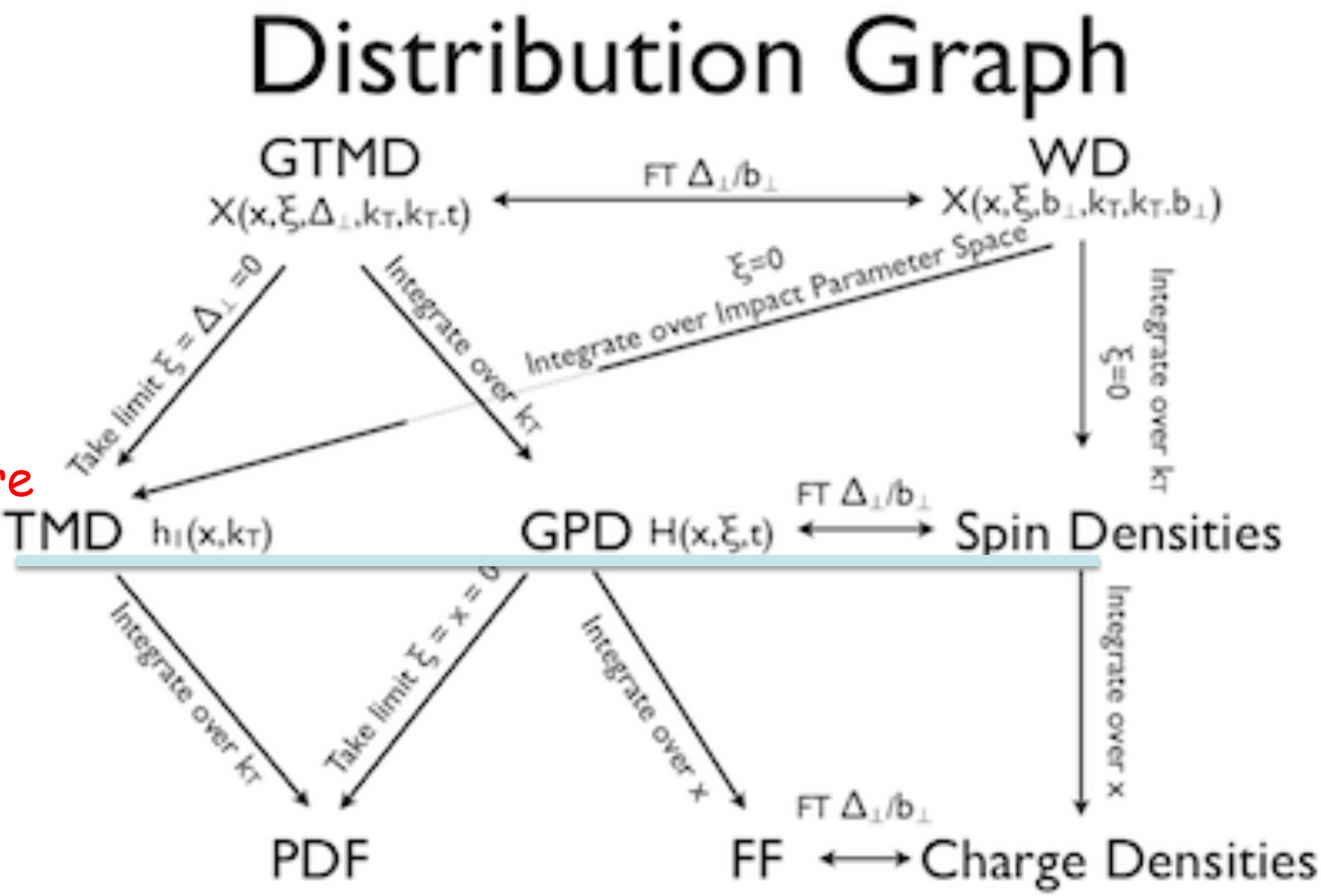
$h_1^{\perp(0)}(x)$

Is this more general than R✖Dq or simpler quark models?

Wigner distributions or Generalized TMDs?

What happens at the unintegrated level

we are here



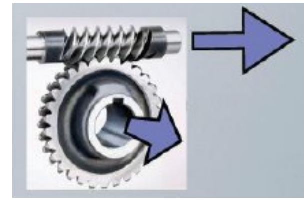
M. Murray

Worm gear I

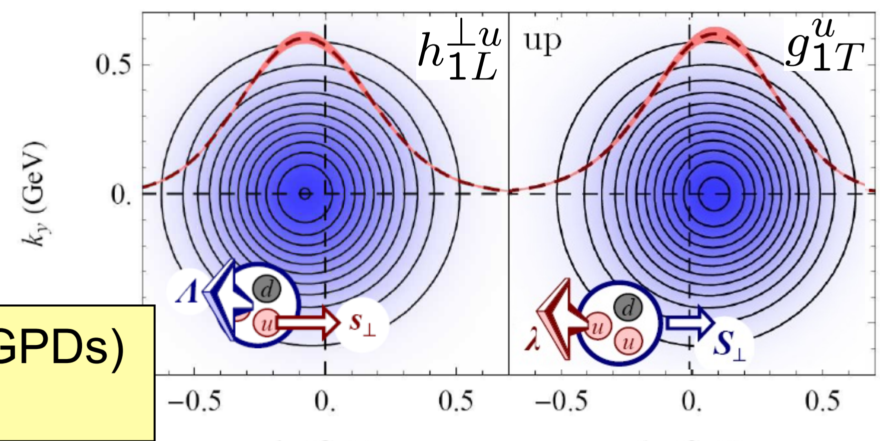
N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

pretzelosity

Worm gear II



$$g_{1T}^{u} \approx -h_{1L}^{\perp u} \quad ?$$



Worm gear TMDs are unique (no analog in GPDs)
Is that certain? How to check?

B.Musch arXiv:0907.2381
B.Pasquini et al, arXiv:0910.1677
H. Avakian, Frascati, Oct 21

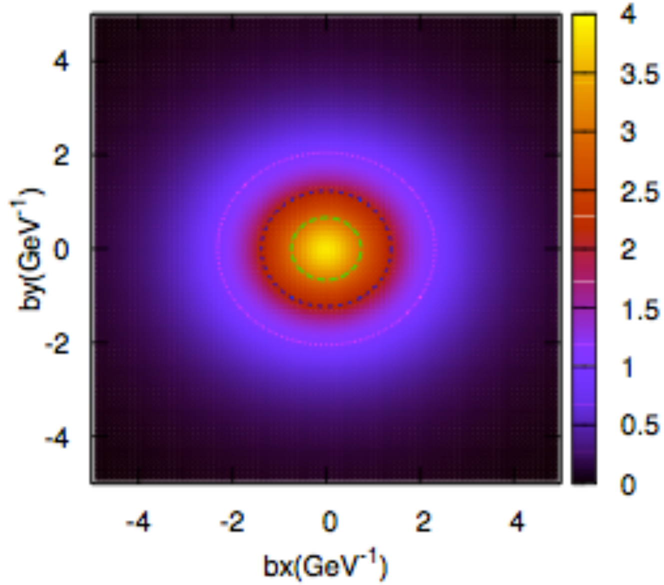
Wigner distribution studies

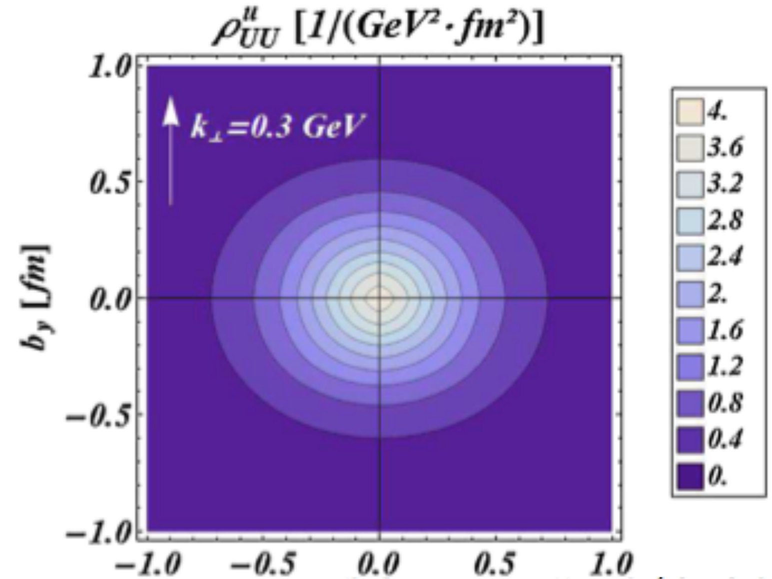
Gonzalez, Goldstein, S.L., R-Diquark Model

Lorce, Pasquini, (2011) LCCQM

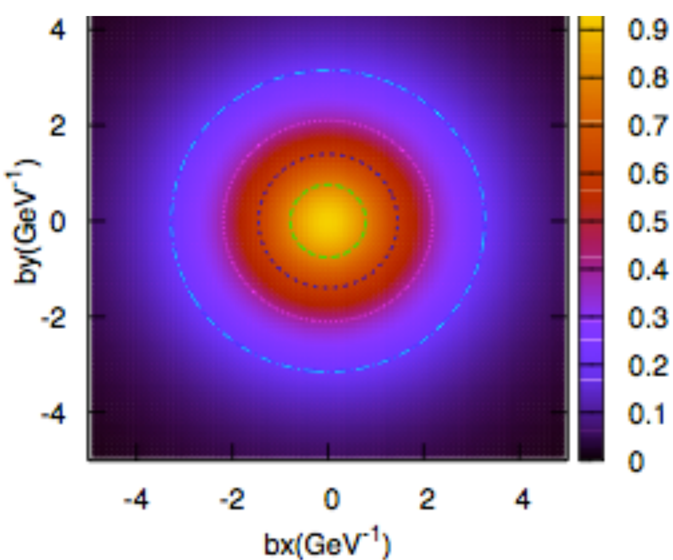
H

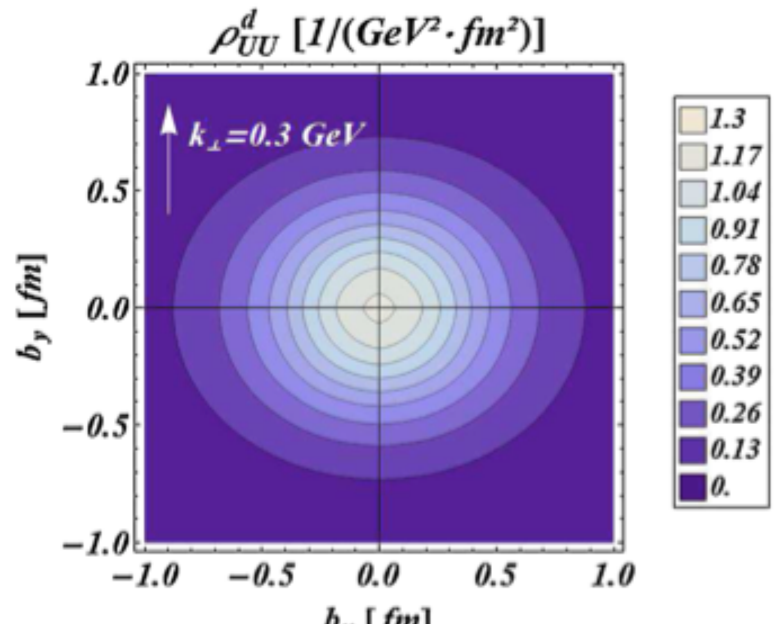
H_u [1/GeV^4]





H_d [1/GeV^4]





Spin Analogs 8 →8

N/q	U	L	T
U	$\mathbf{f_1}$		$\mathbf{h_1^\perp}$
L		$\mathbf{g_1}$	$\mathbf{h_{1L}^\perp}$
T	$\mathbf{f_{1T}^\perp}$	$\mathbf{g_{1T}^\perp}$	$\mathbf{h_1 h_{1T}^\perp}$

Connecting GTMDs
16 complex
Only 8 real combinations here

N,N'/q,q'	U	L	T(Xor Y)
U	H		$2(H^{\sim}_T)+E_T$
L		$H^{\sim}$	$E^{\sim}_T$
T	E	$E^{\sim}$	H_T , $H^{\sim}_T$

TMDs & GPDs

- $f_1 \quad \propto A{++,++} + A{+-,+-} + A{--,--} + A{-+,-+}$

  $= A^{TY}_{++,++} + A^{TY}_{+-,+-} + A^{TY}_{--,--} + A^{TY}_{-+,-+} \sim H$
- $g_{1L} \propto A{++,++} - A{+-,+-} + A{--,--} - A{-+,-+}$

  $= A^{TY}_{++,--} + A^{TY}_{+-,+-} + A^{TY}_{--,++} + A^{TY}_{-+,-+} \sim H^{\sim}$
- $h_{1T}^{\perp} \propto A_{+-,-+} + A_{-+,+-} \sim H_T^{\sim}$ mixture of T_Y & T_X
- "T"-odd TMD vs. GPD
- $f_{1T}^{\perp} \propto A^{TY}_{++,++} + A^{TY}_{+-,+-} - A^{TY}_{--,--} - A^{TY}_{-+,-+} \sim E$
- $h_1^{\perp} \propto A^{TY}_{++,++} - A^{TY}_{+-,+-} + A^{TY}_{--,--} + A^{TY}_{-+,-+} \sim 2H_T^{\sim} + E_T$

  etc.

GTMDs

Meissner, Metz & Schlegel JHEP 08, 056 (2009).

a. define GTMDs:

$$W^{[\Gamma]}_{\lambda\lambda'}(P,x,\vec{k}_T,\Delta,N;\eta) = \int dk^- \, W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta)$$

$$= \frac{1}{2}\int \frac{dz^- \, d^2\vec{z}_T}{(2\pi)^3} \, e^{ik\cdot z} \, \langle p',\lambda'|\, \bar{\psi}\Big(-\frac{1}{2}z\Big)\, \Gamma\, \mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,|\,n\Big)\, \psi\Big(\frac{1}{2}z\Big)\, |p,\lambda\rangle \Big|_{z^+=0} .$$

b. General decomposition into Lorentz-Dirac structures: 16 complex valued functions, but ½ have parity odd prefactors → 8 functions for completeness

$$W^{[\gamma^+]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{1,4}\Big]u(p,\lambda)\,, \qquad (3.6)$$

$$W^{[\gamma^+\gamma_5]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon_T^{ij}k_T^i\Delta_T^j}{M^2}G_{1,1} + \frac{i\sigma^{i+}\gamma_5 k_T^i}{P^+}G_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta_T^i}{P^+}G_{1,3} + i\sigma^{+-}\gamma_5\,G_{1,4}\Big]u(p,\lambda)\,, \qquad (3.7)$$

$$W^{[i\sigma^{j+}\gamma_5]}_{\lambda\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\Big[-\frac{i\varepsilon_T^{ij}k_T^i}{M}H_{1,1} - \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}H_{1,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}H_{1,3} + \frac{k_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}H_{1,4} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}H_{1,5} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5\Delta_T^k}{M\,P^+}H_{1,6} + \frac{k_T^j\,i\sigma^{+-}\gamma_5}{M}H_{1,7} + \frac{\Delta_T^j\,i\sigma^{+-}\gamma_5}{M}H_{1,8}\Big]u(p,\lambda)\,. \qquad (3.8)$$

Projecting GTMDs

c. ξ=0 and Δ_T=0 for TMDs

or integrate k_T for GPDs

Meissner, Metz & Schlegel

TMDs:

$$f_1(x,\vec{k}_T^2) = F^e_{1,1}(x,0,\vec{k}_T^2,0,0)\,,$$
$$f_{1T}^{\perp}(x,\vec{k}_T^2;\eta) = -F^o_{1,2}(x,0,\vec{k}_T^2,0,0;\eta)\,,$$
$$g_{1L}(x,\vec{k}_T^2) = G^e_{1,4}(x,0,\vec{k}_T^2,0,0)\,,$$
$$g_{1T}(x,\vec{k}_T^2) = G^e_{1,2}(x,0,\vec{k}_T^2,0,0)\,,$$
$$h_1^{\perp}(x,\vec{k}_T^2;\eta) = -H^o_{1,1}(x,0,\vec{k}_T^2,0,0;\eta)\,,$$
$$h_{1L}^{\perp}(x,\vec{k}_T^2) = H^e_{1,7}(x,0,\vec{k}_T^2,0,0)\,,$$
$$h_{1T}(x,\vec{k}_T^2) = H^e_{1,3}(x,0,\vec{k}_T^2,0,0)\,,$$
$$h_{1T}^{\perp}(x,\vec{k}_T^2) = H^e_{1,4}(x,0,\vec{k}_T^2,0,0)\,,$$

Chiral odd

“Measurable” GTMDs

- What are relevant GTMDs: Real subset of 16 complex
  after Parity & T-reversal on functions of
  (x, ξ, k_T^2, $k_T \cdot \Delta_T$, Δ_T^2) and Hermiticity.
  c.f. $A_{cd,ab}=(-1)^{c-d+a-b}A^*_{-c-d,-a-b}$ leaves 8
- TMDs
- $F_{1,1}^e$, $G_{1,2}^e$, $\mathbf{G_{1,4}^e}$, $H_{1,7}^e$, $\mathbf{H_{1,3}^e}$, $H_{1,4}^e$ & $\mathbf{F_{1,2}^o}$, $\mathbf{H_{1,1}^o}$
- blue shared with GPDs
- GPDs
- $F_{1,2}^e$ with $F_{1,3}^e$, $G_{1,3}^e$, (chiral even)
- $H_{1,1}^e$ with $H_{1,2}^e$, $H_{1,5}^e$ with $H_{1,6}^e$, $H_{1,8}^e$ (chiral odd)
- Total of <u>13 real GTMDs</u> will give 8TMDs+8GPDs for
- $\xi=0$ and $\Delta_T=0$ or integrate k_T
- Applying GTMDs: $F_{1,2}^e$, $F_{1,4}^e$, $G_{1,1}^e$, $G_{1,4}^e$
  (Lorce & Pasquini) relate to ρ_{UU} ρ_{LU} ρ_{UL} ρ_{LL}

GTMD models

- Applying GTMDs: $F_{1,2}^{e}$, $F_{1,4}^{e}$, $G_{1,1}^{e}$, $G_{1,4}^{e}$ (Lorce & Pasquini) relate to ρ_{UU} ρ_{LU} ρ_{UL} ρ_{LL}

But $F_{1,4}^{e}$, $G_{1,1}^{e}$ have no measureable equivalent since LU and UL would be single longitudinal spin asymmetries.

- trans odd $f_{1T}^{\perp}$ and $h_1^{\perp}$ both related to odd GTMDs **$F_{1,2}^{o}$, $H_{1,1}^{o}$**

while spin-similar GPDs E & ($2H_T$-tilde + E_T) involve only even GTMDs.

- Applying **R✖**Dq to GTMDs & TMDs
- How does Regge in **R✖**Dq enter? Consider inclusive scattering & generalized optical theorem to relate SIDIS to Im part of 3-body elastic forward scattering

Observables

- Restrictions on what is measurable in SIDIS & exclusives
- SSAs limited by Parity to be normal to scattering plane – *all* that can be known
- Cannot know how much is "sideways" polarized
- Wigner distributions are *not* measurable [k_x , b_x]. . .
- GTMDs (Meissner, Metz, Goeke &/or Schlegel) extra nomenclature for unintegrated model of GPDs

Conclusions

- Flexible Model GPDs → phenomenology (DVCS & DVMP)
- Spectator models relate Chiral even to Chiral odd GPDs. How far broken?

  Regge behavior R✖Dq
- Exclusive π^0 electroproduction observables involve chiral odd GPDs

  $d\sigma_T/dt$, $d\sigma_{TT}/dt$, A_{UT}, beam asymmetry, beam-target correlations,

  $d\sigma_L/dt$, $d\sigma_{LT}/dt$
- GPD ⇔ TMDs through transversity & R✖Dq
- Subset of Wigner distribution/GTMDs are accessed via models

  Does this facilitate study of OAM?

  Can 3-d imaging be completely model independent?