Holographic hydrodynamics of systems with broken rotational symmetry

Johanna Erdmenger

Max-Planck-Institut für Physik, München

Based on joint work with M. Ammon, V. Grass, M. Kaminski, P. Kerner, H.T. Ngo, A. O'Bannon, H. Zeller

Motivation

J.E., Haack, Kaminski, Yarom 0809.2488 (JHEP);

Banerjee, Bhattacharya, Bhattacharyya, , Loganayagam, Dutta, Surowka 0809.2596 (JHEP):

Gauge/Gravity Duality at finite charge density requires
5d Chern-Simons term:

Axial contribution to hydrodynamic expansion of current

$$J_\mu = \rho u_\mu + \xi \omega_\mu, \quad \omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho} u^\nu \partial^\sigma u^\rho$$

Chiral vortical effect in field-theory context

Related to axial anomaly (Son, Surowka 2009)

Motivation:

Gauge/gravity duality: New tools for strongly coupled systems

Famous result: Shear viscosity/Entropy density

Kovtun, Son, Starinets

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

From `Planckian time' $\tau_P = \frac{\hbar}{k_B T}$, Universal result

This talk:
Deviations from this result at leading order in λ and N

Holographic proof of universality relies on space-time isotropy

Key ingredient for changes to the universal result: Spacetime anisotropy

Rotational invariance broken

Holographic p-wave superfluids/superconductors

ρ meson condensate breaks rotational symmetry

At finite isospin density (or in external magnetic field)

Outline

- Holographic superconductors
- Transport coefficients in anisotropic systems
- Condensates at finite magnetic field

Holographic Superfluids

- Holographic Superfluids from charged scalar in Einstein-Maxwell gravity (Gubser; Hartnoll, Herzog, Horowitz)
- p-wave superfluid
  Current dual to gauge field condensing (Gubser, Pufu)
  SU(2) Einstein-Yang-Mills model

s-wave superfluid:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

Operator $\mathcal{O}$ dual to scalar ψ condensing

Herzog, Hartnoll, Horowitz 2008

p-wave superfluid:

$$S = \frac{1}{2\kappa^2}\int d^4x \left[R - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{6}{L^2}\right]$$

Current J^1_x dual to gauge field component A^{1x} condensing

Gubser, Pufu 2008

P-wave superfluid from probe branes

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

- A holographic superconductor with field theory in 3+1 dimensions for which
- the dual field theory is explicitly known
- there is a qualitative ten-dimensional string theory picture of condensation

This is achieved in the context of
adding flavour to gauge/gravity duality

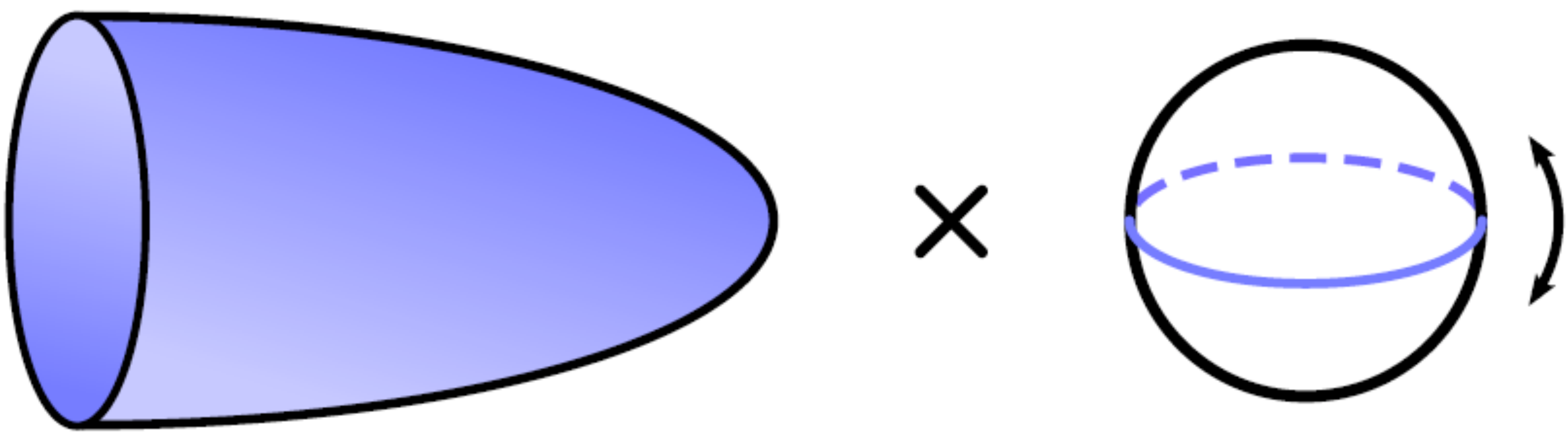
Brane probes added on gravity side $\Rightarrow$
fundamental d.o.f. in the dual field theory (quarks)

Additional D-branes within $AdS_5 \times S^5$ or deformed version thereof

Quarks within Gauge/Gravity Duality

Adding D7-Brane Probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



Probe brane fluctuations $\Rightarrow$ Masses of mesons ($\bar{\psi}\psi$ bound states)

On gravity side:
Probe brane fluctuations described by Dirac-Born-Infeld action

$$S_{\mathrm{DBI}} = -T_{D7} \int \mathrm{d}^8\xi \, \mathrm{Str}\sqrt{|\det(G + 2\pi\alpha' F)|}$$

On field theory side: Lagrangian explicitly known

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}({\psi_q}^i, {\phi_q}^i)$$

Turn on finite temperature and isospin chemical potential:

Finite temperature: Embed D7 brane in black hole background

Isospin chemical potential: Probe of two coincident D7 branes

Additional symmetry U(2) = SU(2)$_I$ x U(1)$_B$

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \ldots, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \ldots$$

Condensate $\langle J_3 \rangle, \quad J_3 = \bar{\psi}_d \gamma_3 \psi_u + \ bosons$

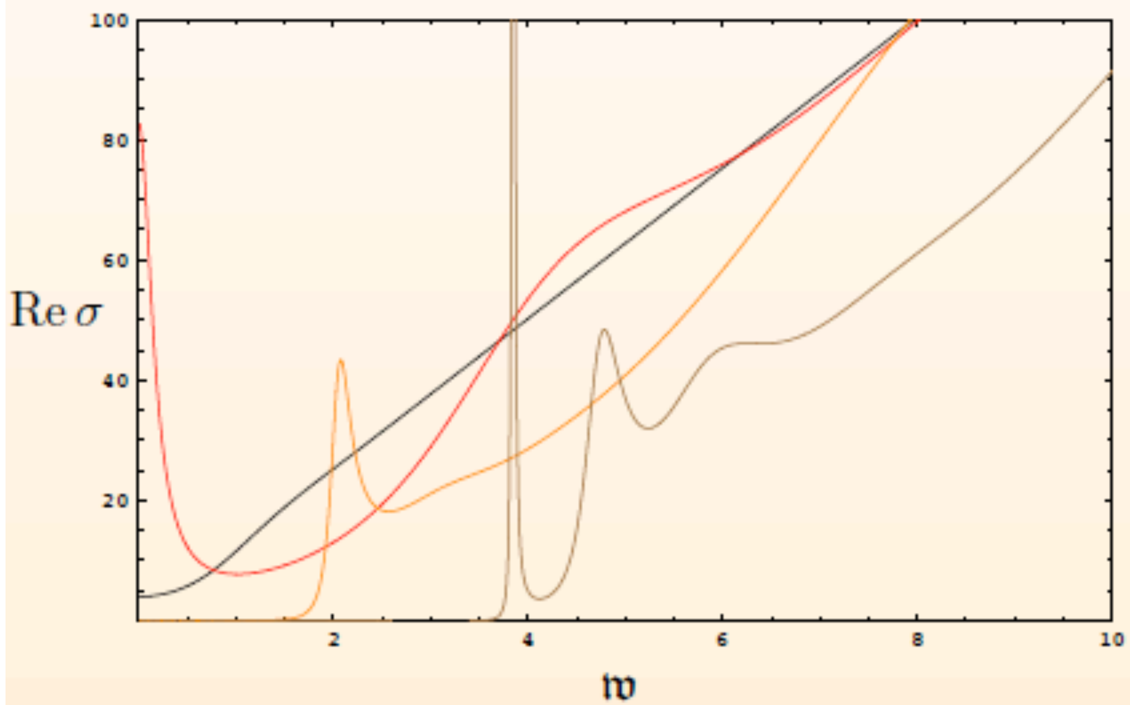
rho meson condensation in Sakai-Sugimoto:
Aharony, Peeters, Sonnenschein, Zamaklar '07

Calculate correlators from fluctuations

Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



Ammon, J.E., Kaminski, Kerner '08

$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma

Effective 5d model ➡ anisotropic shear viscosity

Bottom-up: Including the backreaction

Ammon, J.E., Graß, Kerner, O'Bannon 0912.3515

Einstein-Yang-Mills-Theory with $SU(2)$ gauge group

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right]$$

$$\alpha = \frac{\kappa_5}{\hat{g}}$$

$\alpha^2 \propto$ number of charged d.o.f./all d.o.f.

In presence of $SU(2)$ chemical potential, same condensation process as before

Hairy black hole solution

- metric ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + \frac{r^2}{f(r)^4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

with

$$N(r) = -\frac{2m(r)}{r^2} + r^2$$

AdS boundary $r = r_{\mathrm{bdy}} \to \infty$ & black hole horizon $r = r_h$

- gauge field ansatz

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x$$

Field Theory	$\Leftrightarrow$	Gravity
chemical potential μ $SU(2) \to U(1)_3$		$A_t^3 = \phi(r) \neq 0$ $SU(2) \to U(1)_3$
$\langle \mathcal{J}_1^x \rangle \neq 0$ $U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$		$A_x^1 = w(r) \neq 0$ $U(1)_3 \to \mathbb{Z}_2$, $SO(3) \to SO(2)$

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \to \mathbb{Z}_2$ & $SO(3) \to SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

- 5 fields: $\{\phi(r), w(r), \sigma(r), f(r), m(r)\}$

Variation of on-shell action at AdS boundary gives

- energy-momentum tensor

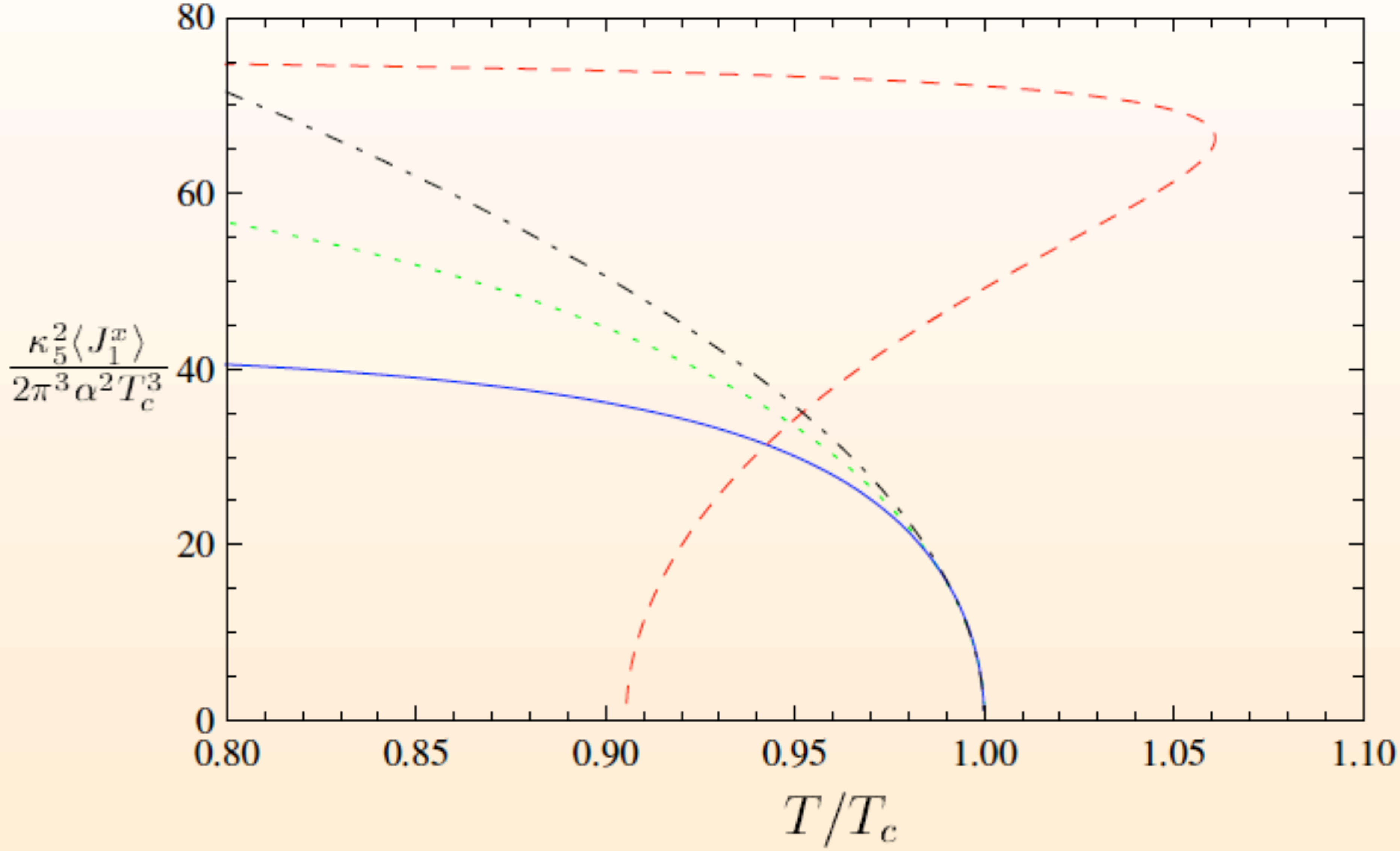
$$\langle \mathcal{T}_{\mu\mu} \rangle \propto T^4 \cdot \mathrm{Func}(m_0^b, f_2^b), \text{ with: } \langle \mathcal{T}_{yy} \rangle = \langle \mathcal{T}_{zz} \rangle \neq \langle \mathcal{T}_{xx} \rangle$$
$$\langle \mathcal{T}_{\mu\nu} \rangle = 0 \text{ for } \mu \neq \nu$$

$$\langle \mathcal{T}_{xx} \rangle = P + \Delta \, \langle \mathcal{J}_1^x \rangle \langle \mathcal{J}_1^x \rangle$$

- condensate

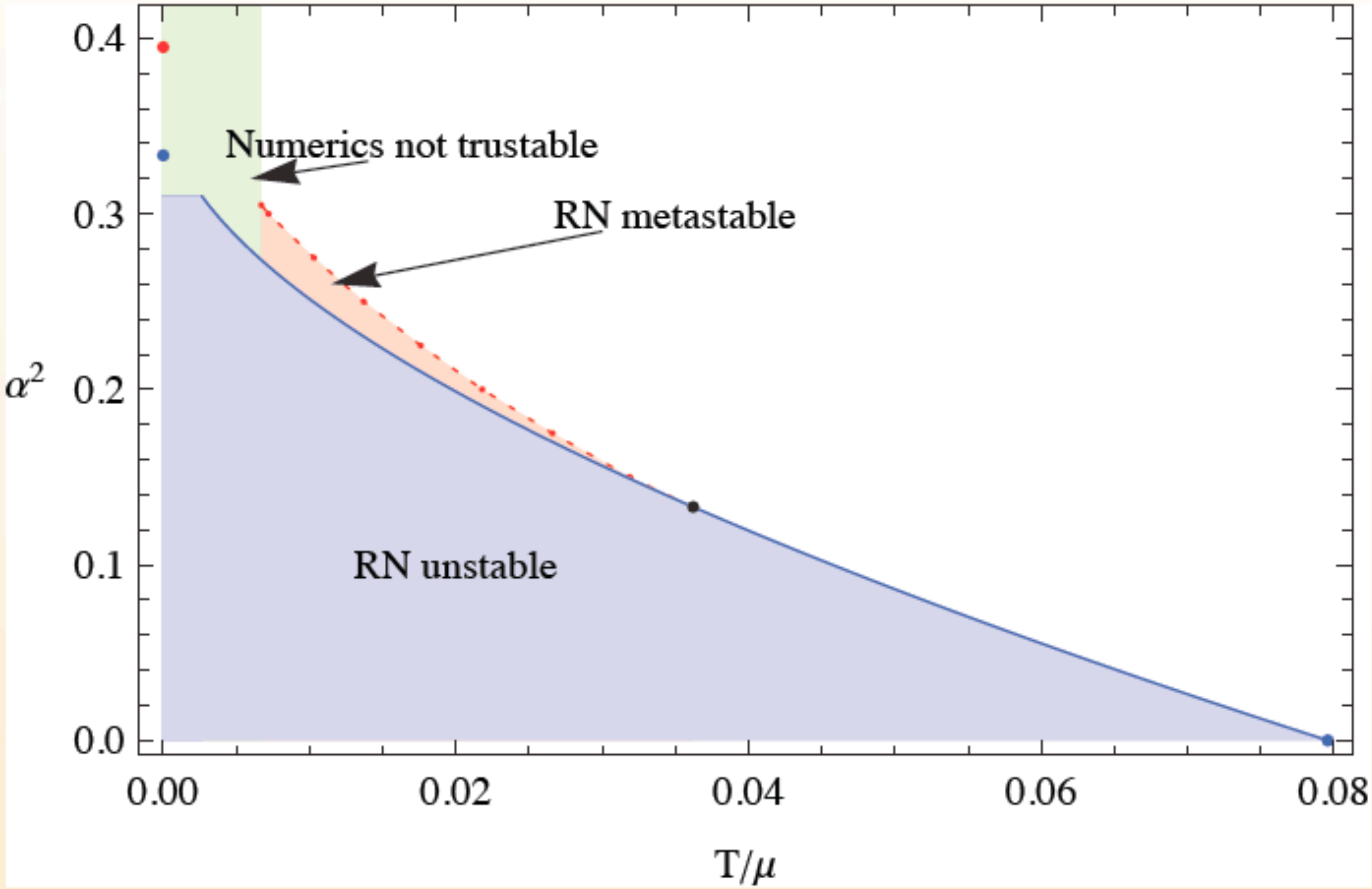
$$\langle \mathcal{J}_1^x \rangle \propto T^3 w_1^b$$

Phase transition



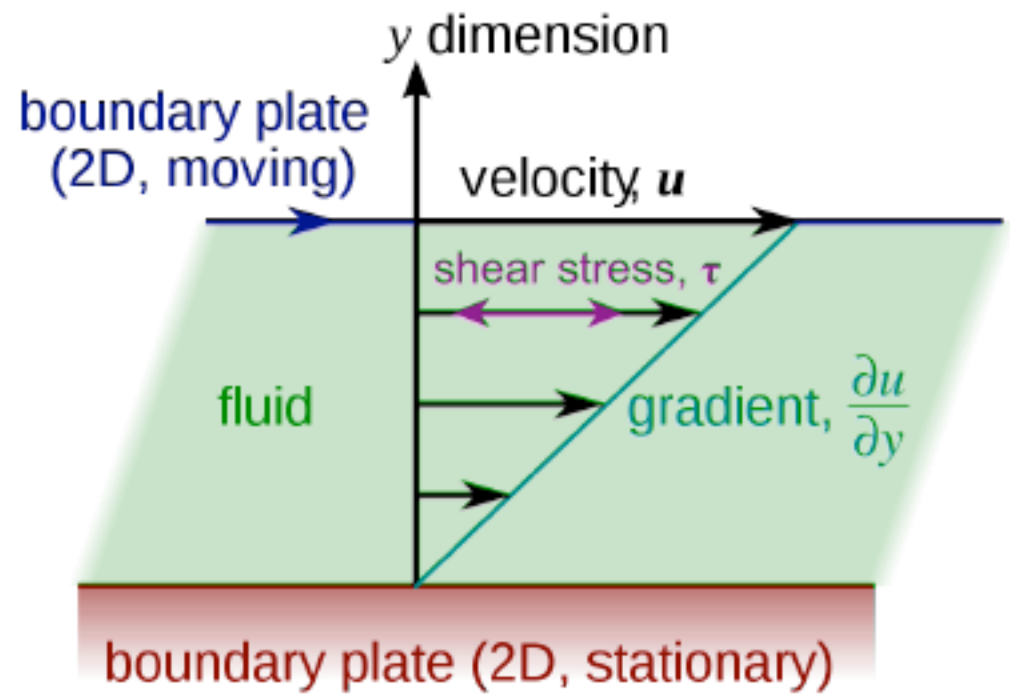
Phase transition becomes first order above α_{crit}

Phase diagram



Anisotropic shear viscosity

- viscosity tensor $\eta_{\alpha\beta\gamma\delta}$
- anisotropic systems:
  21 components
- isotropic systems:
  1 shear viscosity
- transversely isotropic systems:
  2 shear viscosities



Holographic calculation: J.E., Kerner, Zeller 1011.5912; 1110.0007

Classification of Fluctuations

- set $k_\perp = 0$

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around x-axis possible:

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a^a_y$	h_{yr}	4
	$h_{tz}, h_{xz}; a^a_z$	h_{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt}$; a^a_t, a^a_x	$h_{tr}, h_{xr}, h_{rr}; a^a_r$	4

gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 physical modes

Transport coefficients from Green functions

One non-trivial helicity 2 mode ➡

gives well-known result $\eta/s = 1/4\pi$

Helicity 1 modes:

- in $\vec{k} \to 0$ limit additional symmetry:
  $\mathbb{Z}_2$: $x \to -x$, $w \to -w$

$\Rightarrow$ helicity 1 modes decouple in 2 blocks:

even parity: $\{\Psi_t = g^{yy} h_{t\perp}, a^3_\perp, h_{r\perp}\}$

odd parity: $\{\Psi_x = g^{yy} h_{x\perp}, a^1_\perp, a^2_\perp\}$ $\Rightarrow$ 3 independent fields: $\Psi_x, a^1_\perp, a^2_\perp$

$\Rightarrow$ Green's function: 3×3 matrix

Linear response

- choose basis: $a_\perp^\pm = a_\perp^1 \pm \mathrm{i} a_\perp^2$

$\Rightarrow$ transform in fundamental repr. of unbroken $U(1)_3$

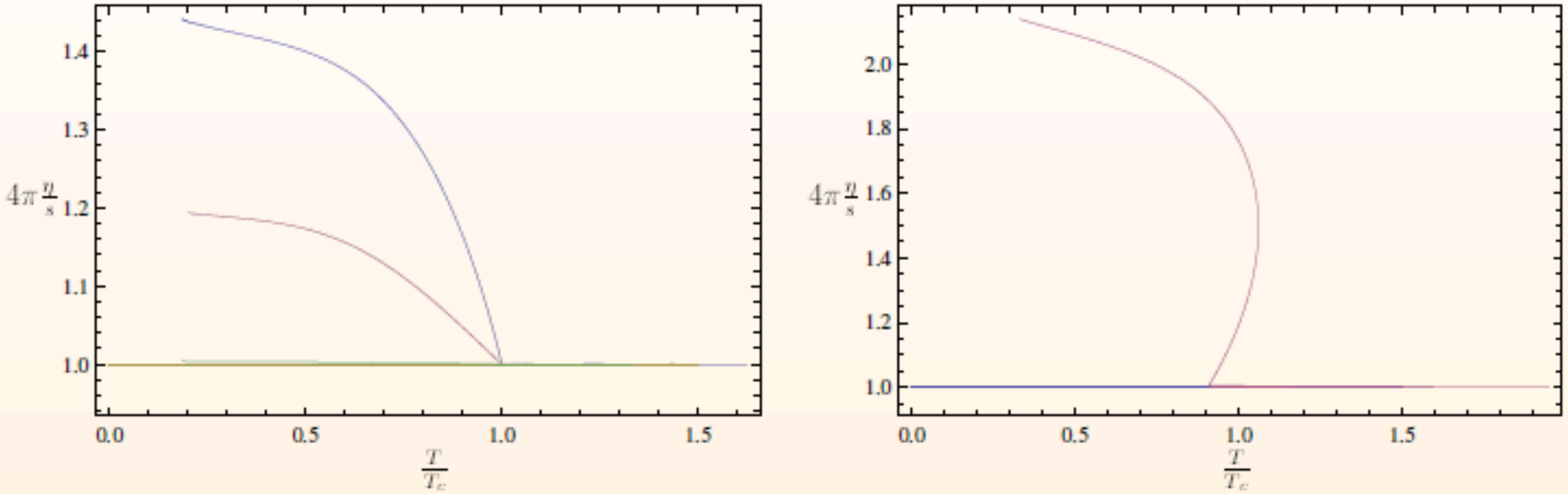
- field theory:

$$\begin{pmatrix} \langle J_+^\perp \rangle \\ \langle J_-^\perp \rangle \\ \langle T^{x\perp} \rangle \end{pmatrix} = \begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_+^{\perp\,x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_-^{\perp\,x\perp} \\ G^{x\perp\,\perp}_{\;\;\;+} & G^{x\perp\,\perp}_{\;\;\;-} & -\langle T_{xx} \rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix} \begin{pmatrix} a_\perp^+ \\ a_\perp^- \\ h_{x\perp} \end{pmatrix}$$

- with

$$\eta_{x\perp} = -\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\left(G^{x\perp,x\perp} \right)$$

Anisotropic shear viscosity



$\eta_{yz}/s = 1/4\pi$; η_{xy}/s dependent on T and on α

Critical behaviour: $1 - 4\pi\frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$, α-independent

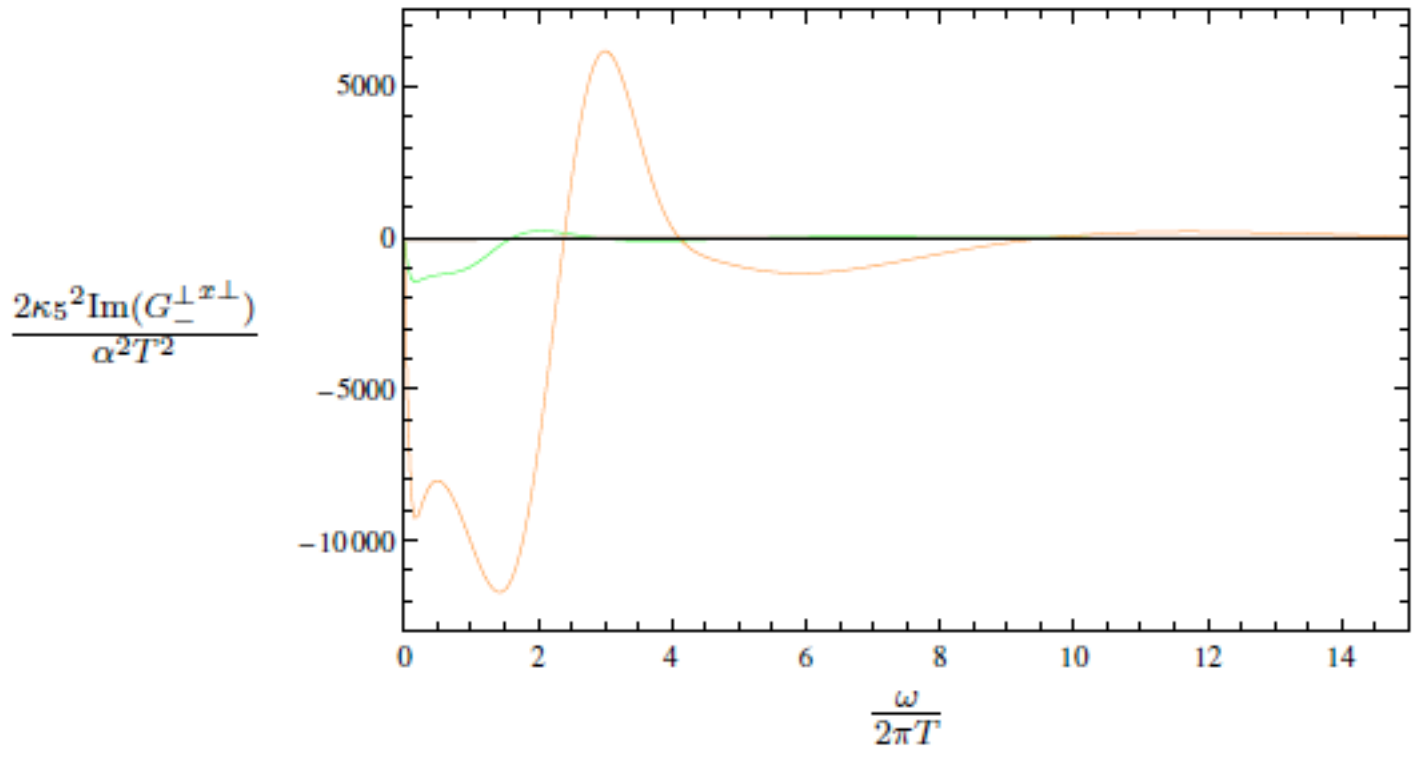
Non-universal behaviour at leading order in λ and N

Critical exponent confirmed analytically in Basu, Oh 1109.4592

Flexoelectric Effect

Nematic phase:
A strain introduces spontaneous electrical polarization

In our case:
A strain $h_{x\perp}$ introduces an inhomogeneity in the current $\mathcal{J}_1^x$ which introduces a current $\mathcal{J}_\pm^\perp$



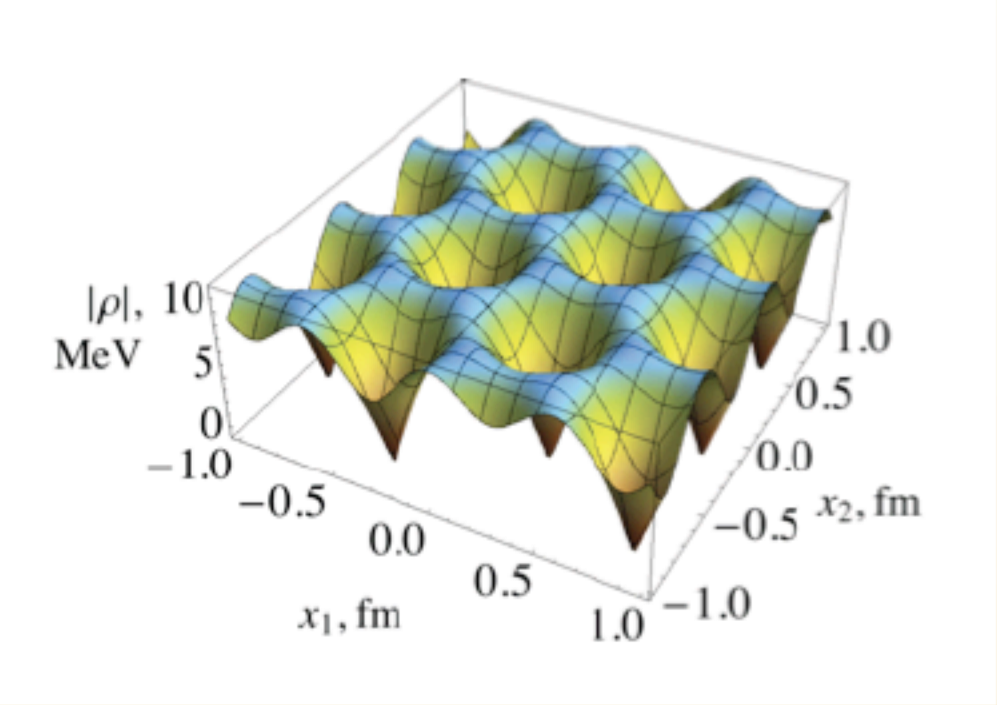
J.E., Kerner, Zeller
1110.0007

External electromagnetic fields

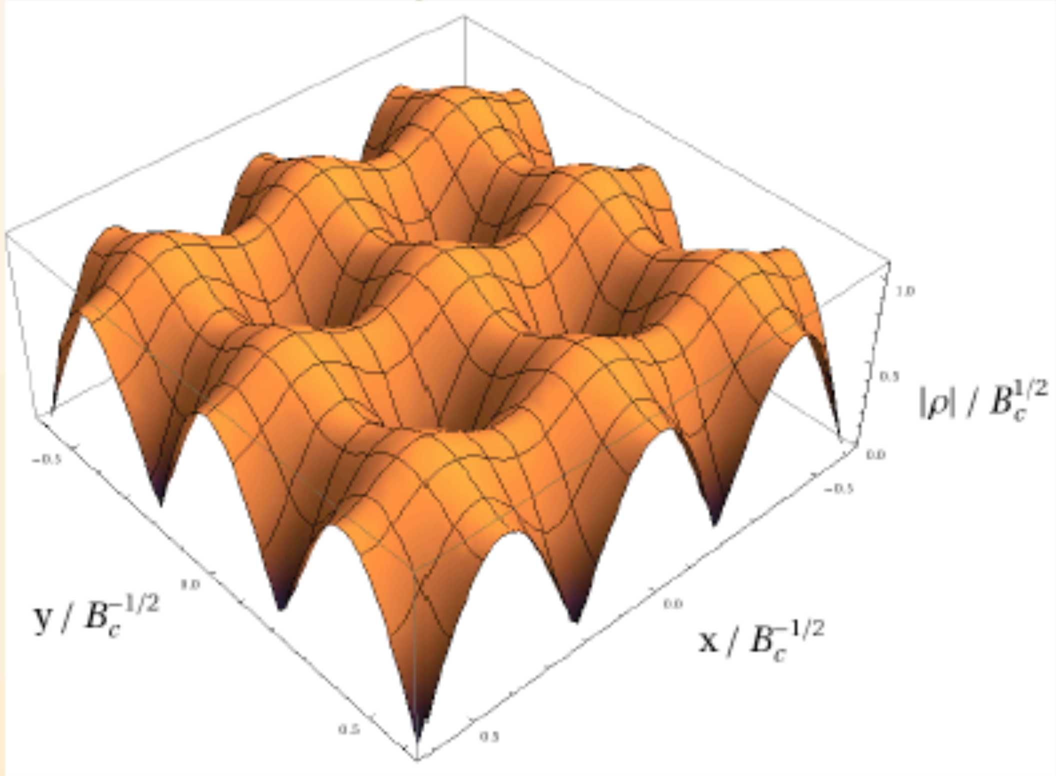
A magnetic field leads to

ρ meson condensation and superconductivity in the QCD vacuum

Effective field theory:
(Chernodub)



Gauge/gravity duality
magnetic field in black hole supergravity background
(J.E., Kerner, Strydom PLB 2011)

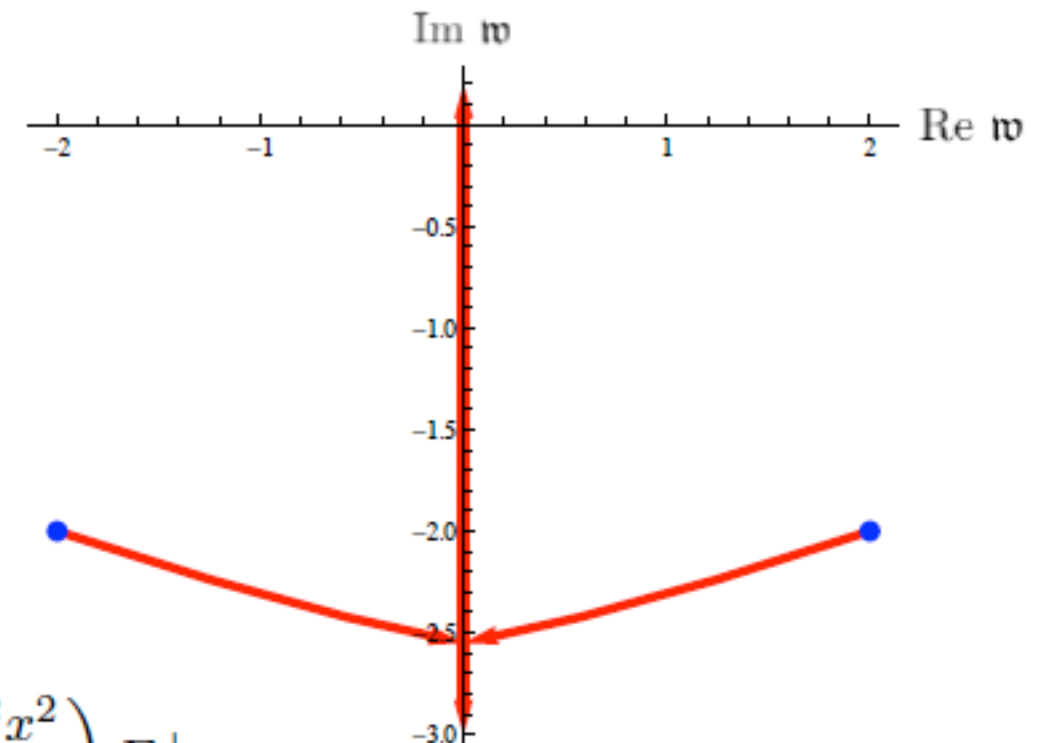


Condensation in magnetic field

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}\left(R-2\Lambda\right) - \frac{1}{4\hat{g}^2}F^a_{\mu\nu}F^{a\mu\nu}\right] + S_{\mathrm{bdy}}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu$$

$$A^3_y = xB$$

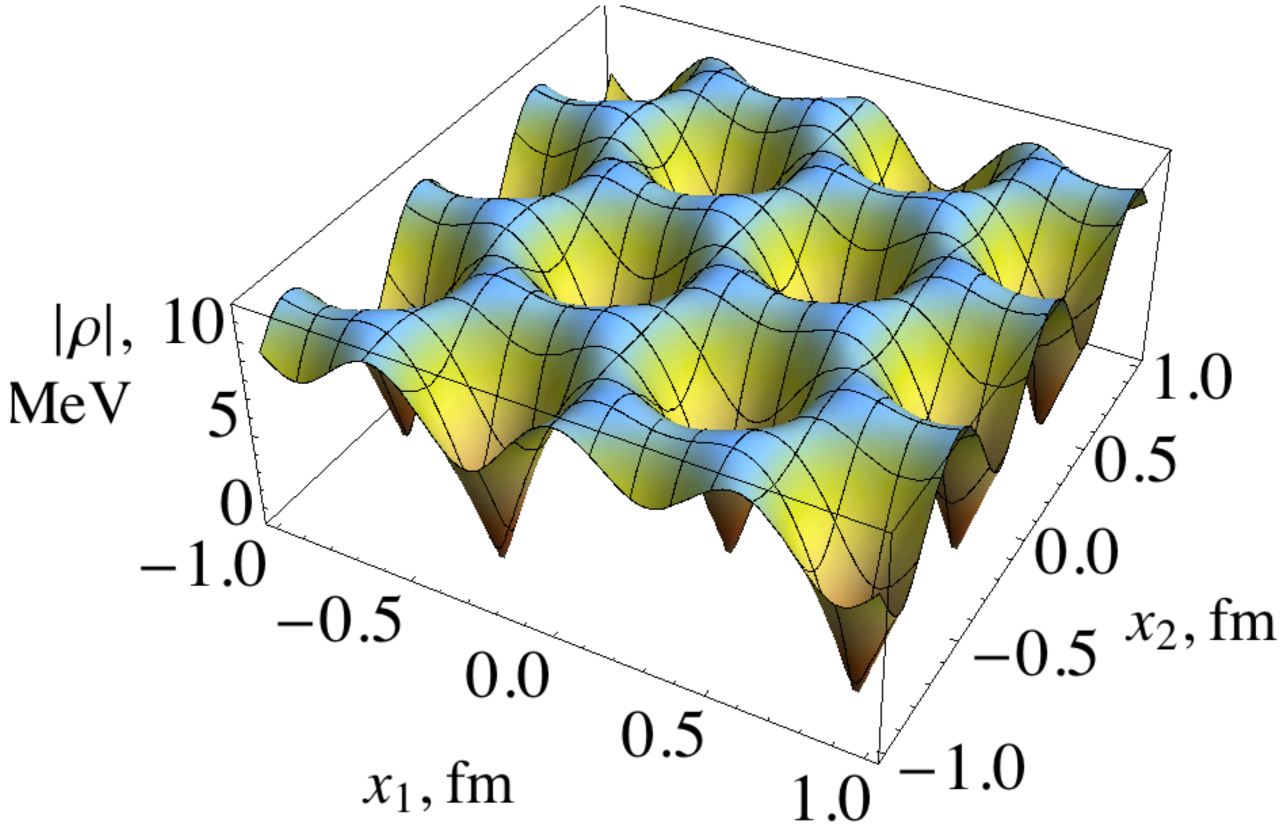


Fluctuations

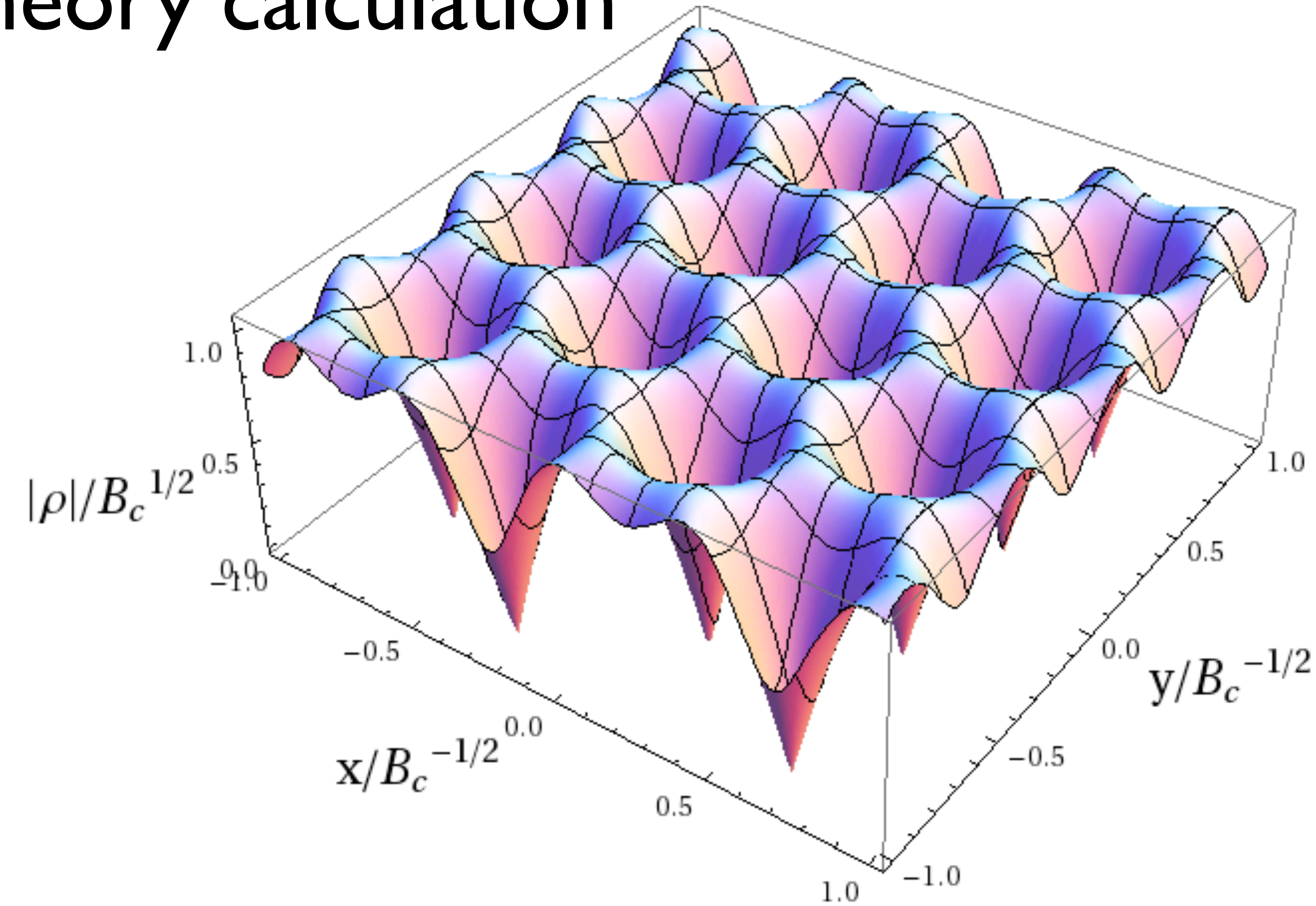
$$0 = \partial_u^2 E_x^+ + \frac{1}{f}\partial_x^2 E_x^+ + \left(\frac{f'}{f} - \frac{1}{u}\right)\partial_u E_x^+ - \frac{2}{xf}\partial_x E_x^+ + \left(\frac{\omega^2}{f^2} - \frac{B^2x^2}{f}\right)E_x^+$$

cf. Chernodub;
Callebaut, Dudas, Verschelde;
Donos, Gauntlett, Pantelidou

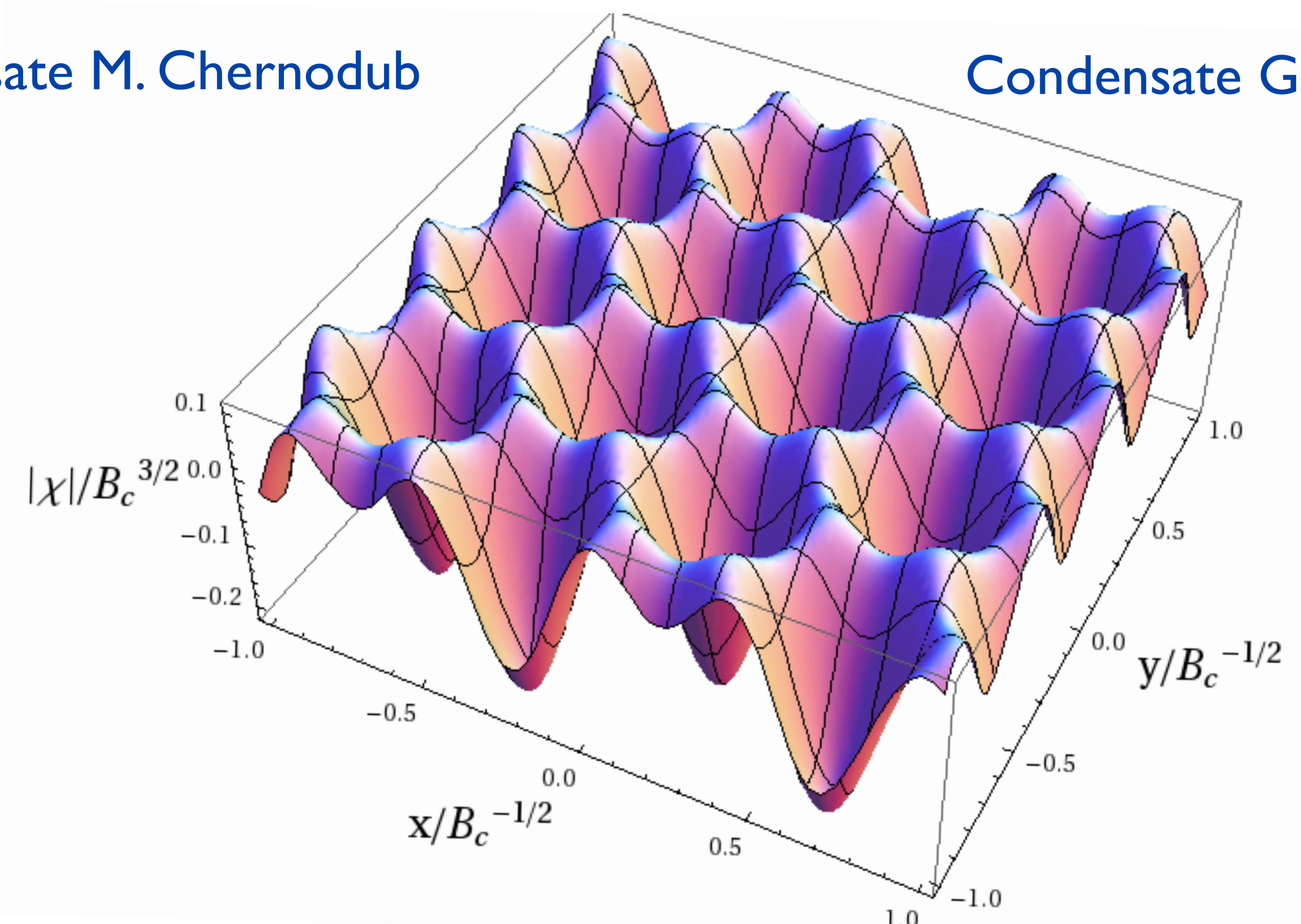
Comparison to field theory calculation



Condensate M. Chernodub



Condensate Gauge/Gravity Duality

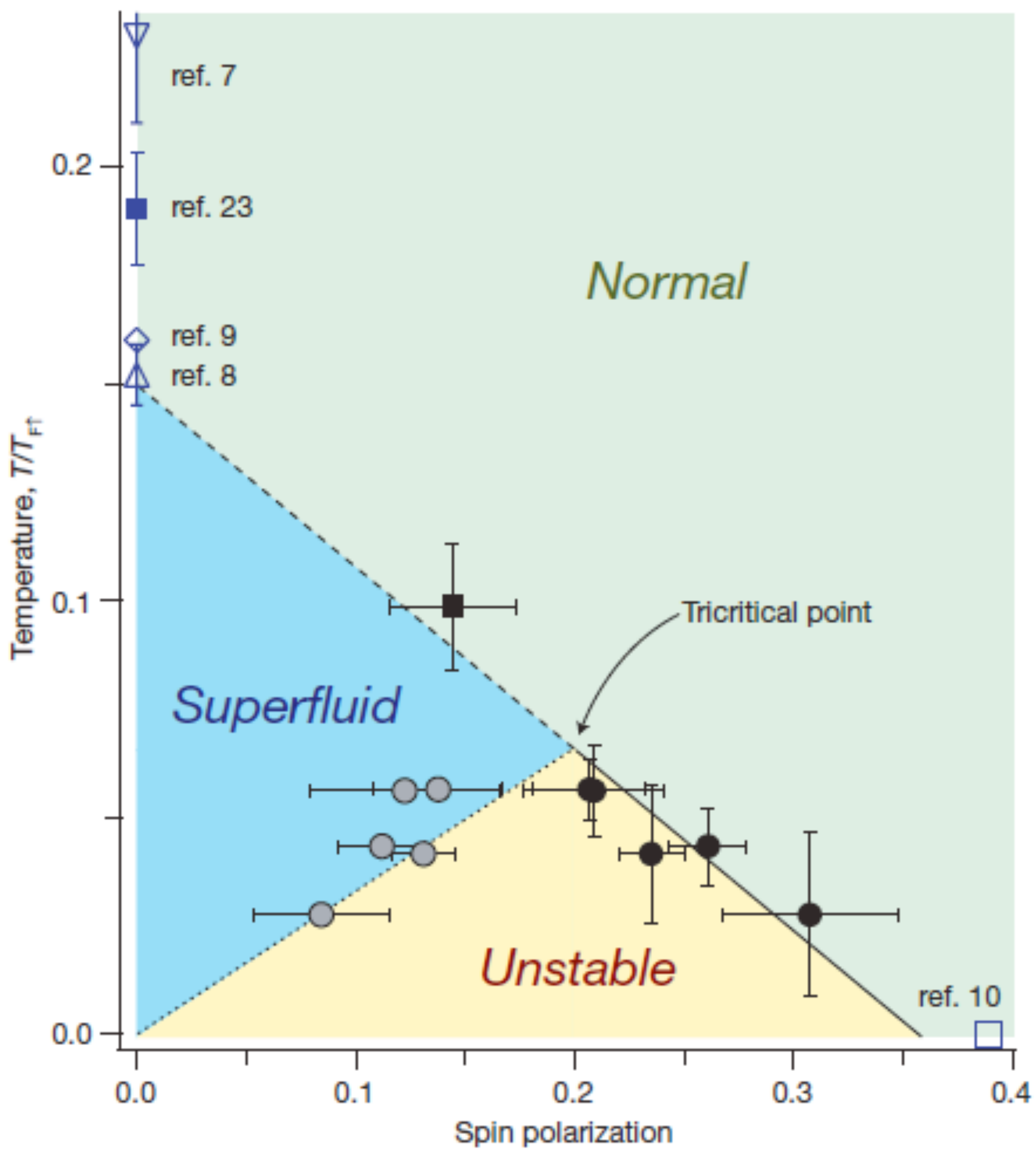


Magnetization Gauge/Gravity Duality

Conclusions

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect
- Condensation at finite magnetic field

Superfluidity in imbalanced mixtures



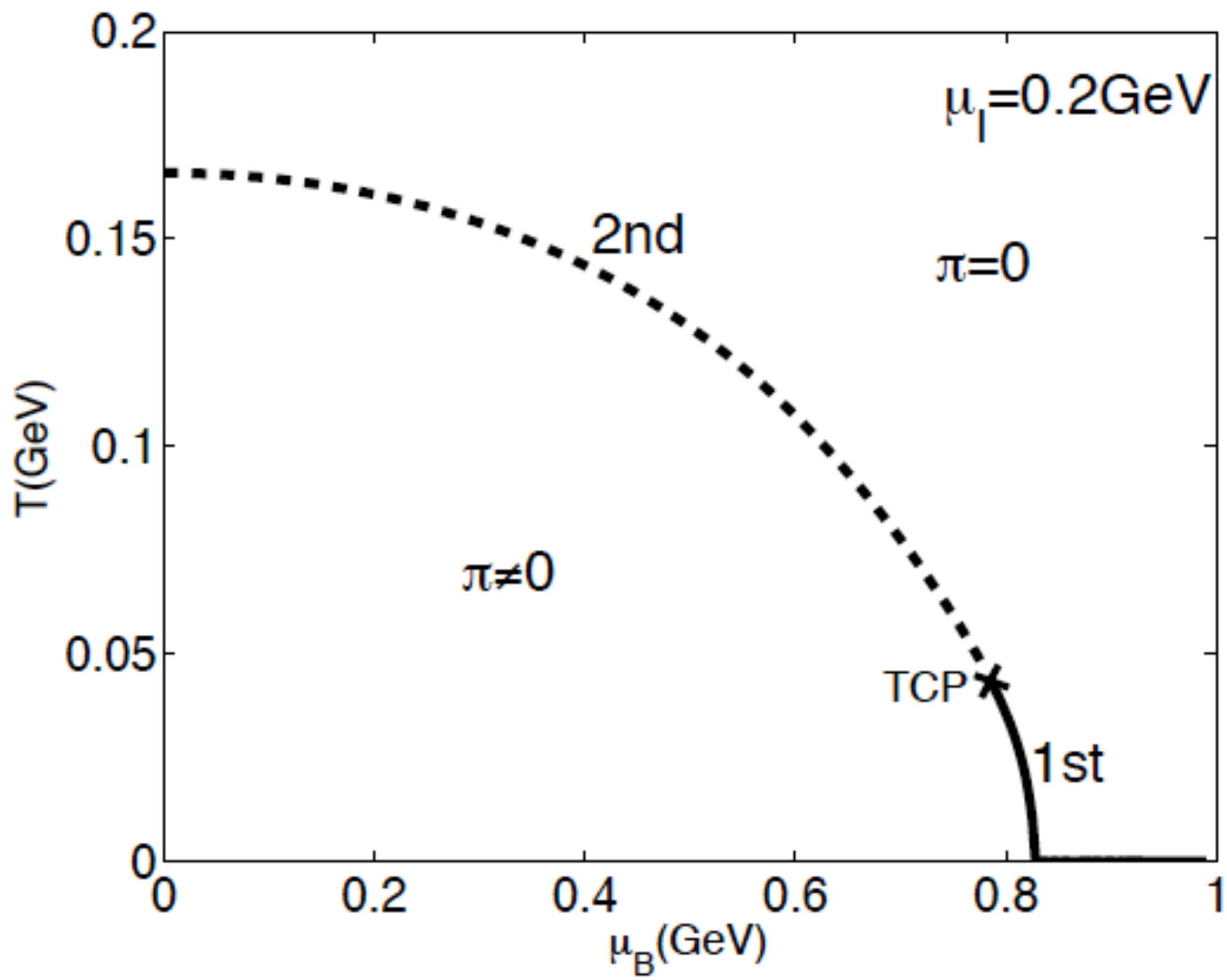
Shin, Schunck, Schirotzek, Ketterle, Nature 2008

Imbalanced mixtures

Contain different number of spin up and spin down particles

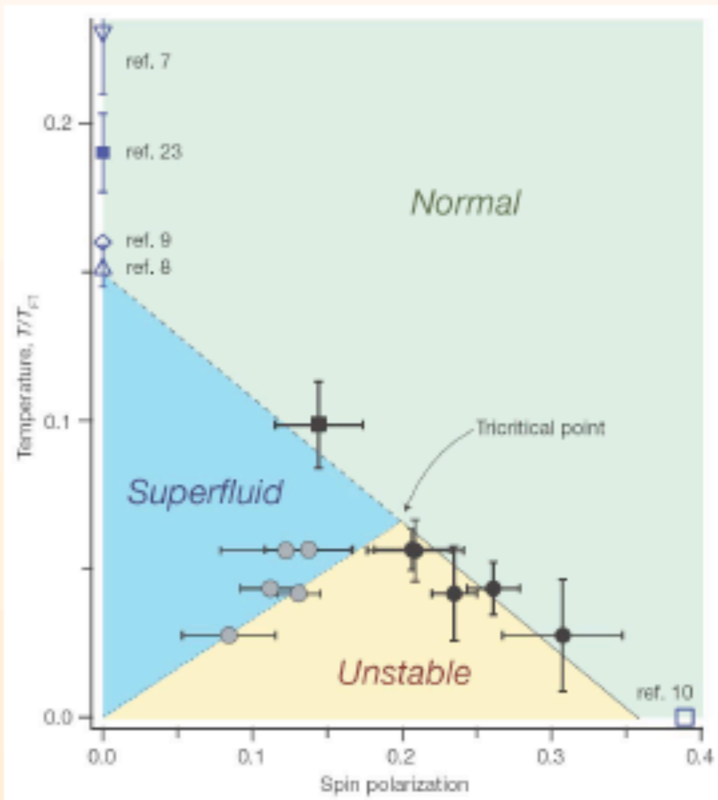
How does an imbalance in numbers (spin polarization) affect the superfluid phase transition?

QCD at finite isospin chemical potential

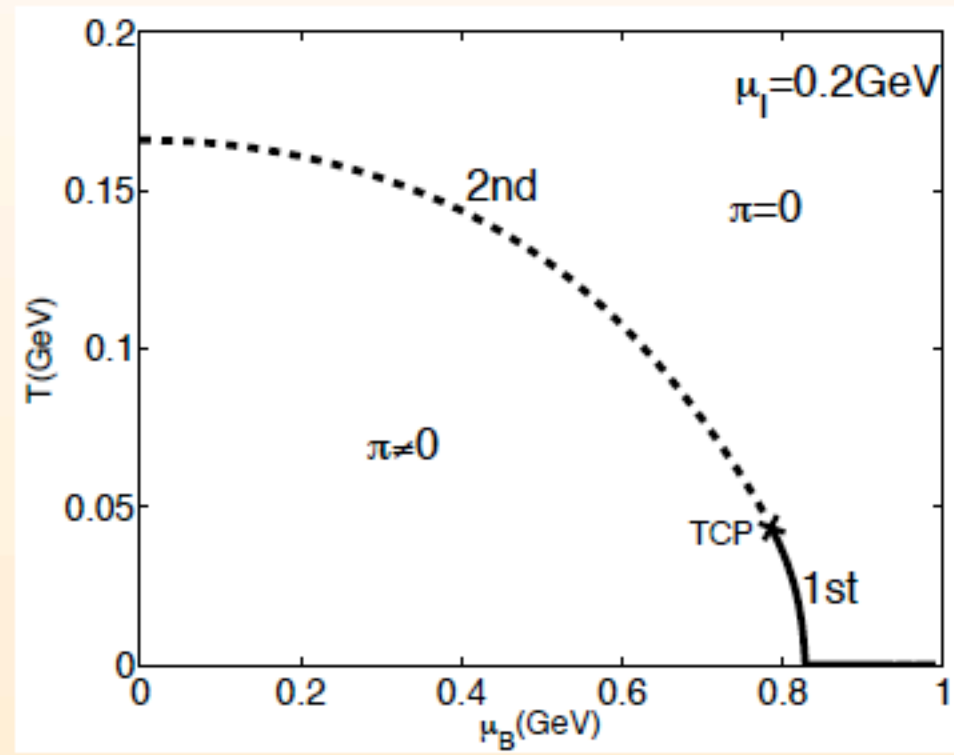


He, Jin, Zhuang, PRD 2005

Inbalanced Mixtures and Quantum Phase Transition



Shin, Schunck, Schirotzek, Ketterle, Nature 2008



He, Jin, Zhuang, PRD 2005

Lithium superfluid

QCD at finite isospin density

There appears to be universal behavior

Can we describe imbalanced mixtures in gauge/gravity duality?

Yes!

Can we obtain a similar phase diagram?

We can, in principle...

Holographic Imbalanced Mixtures

Turn on both isospin and baryon chemical potential

$$U(2) = SU(2)_I \times U(1)_B$$

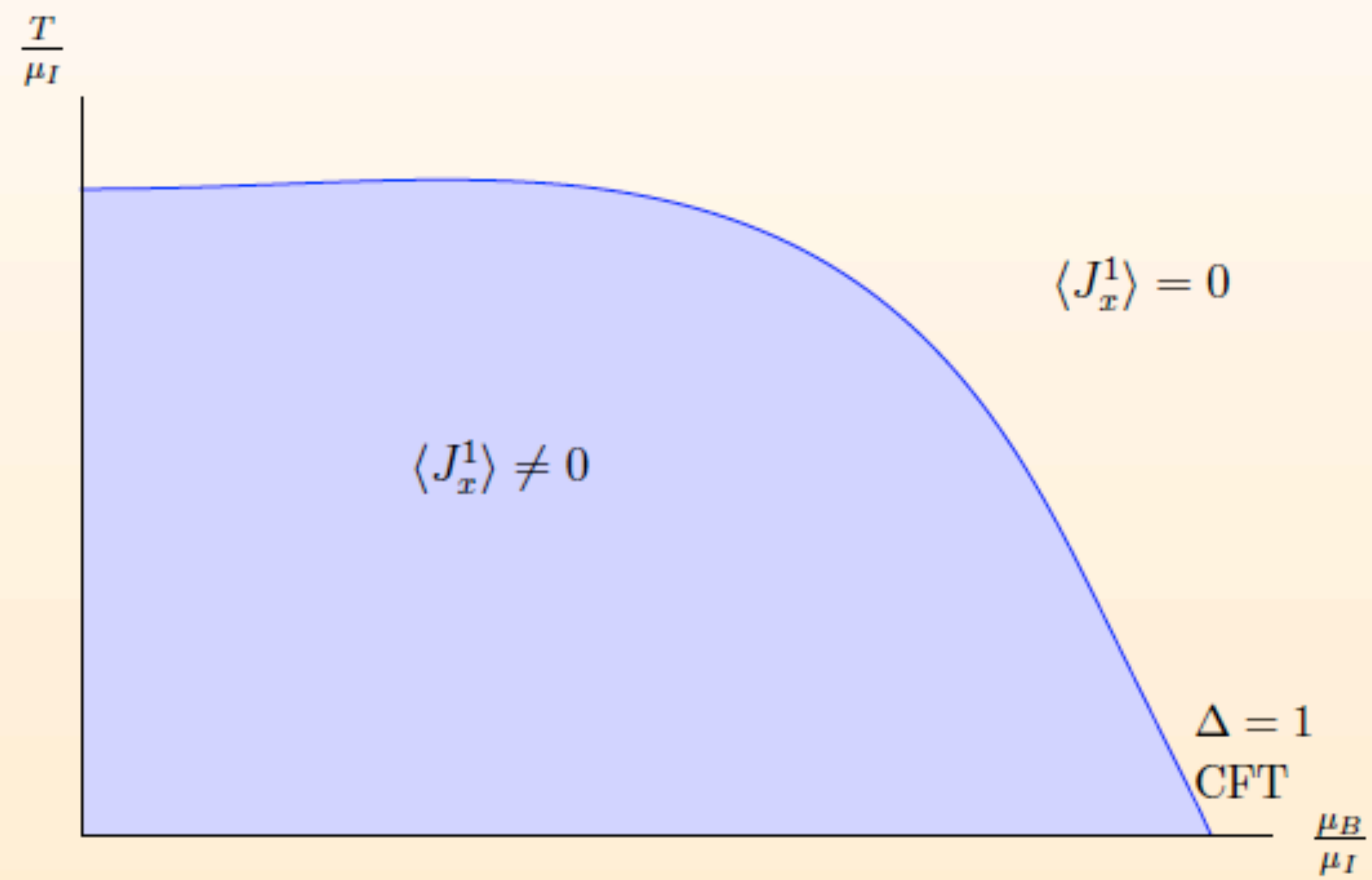
Condensate $\bar{\psi}_d \gamma_3 \psi_u$ (rho meson)

Increasing μ_B turns u into $\bar{u}$ quarks

Inbalanced Mixtures and Quantum Phase Transition

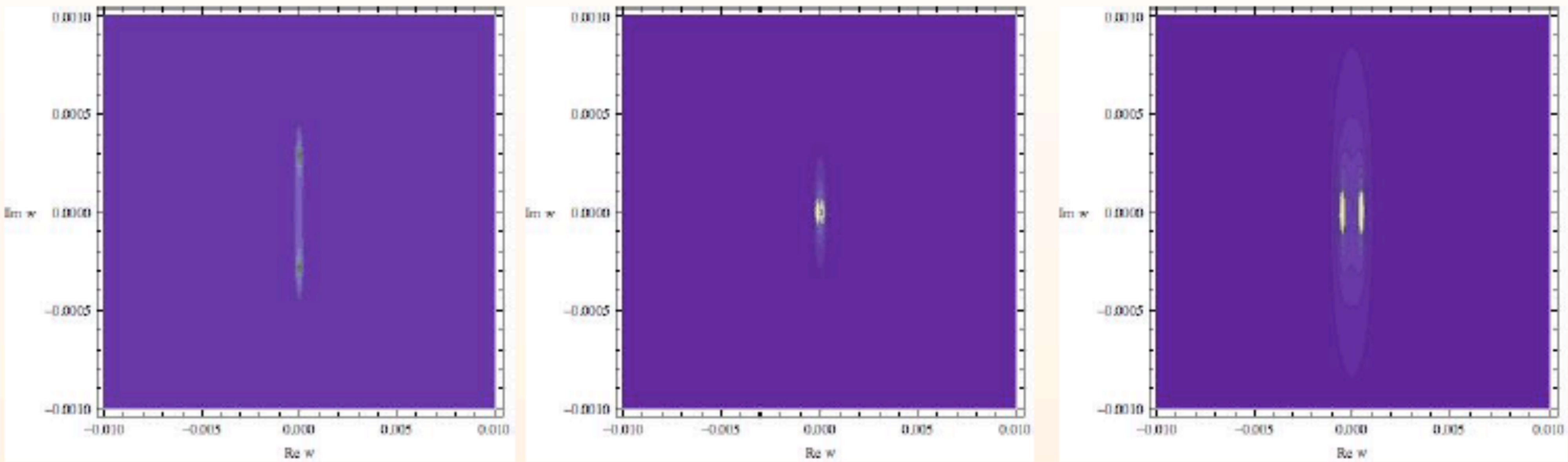
J.E., Graß, Kerner, Ngo 1103.4145

Turn on both isospin and baryon chemical potential in D3/D7 setup



Phase transition second order

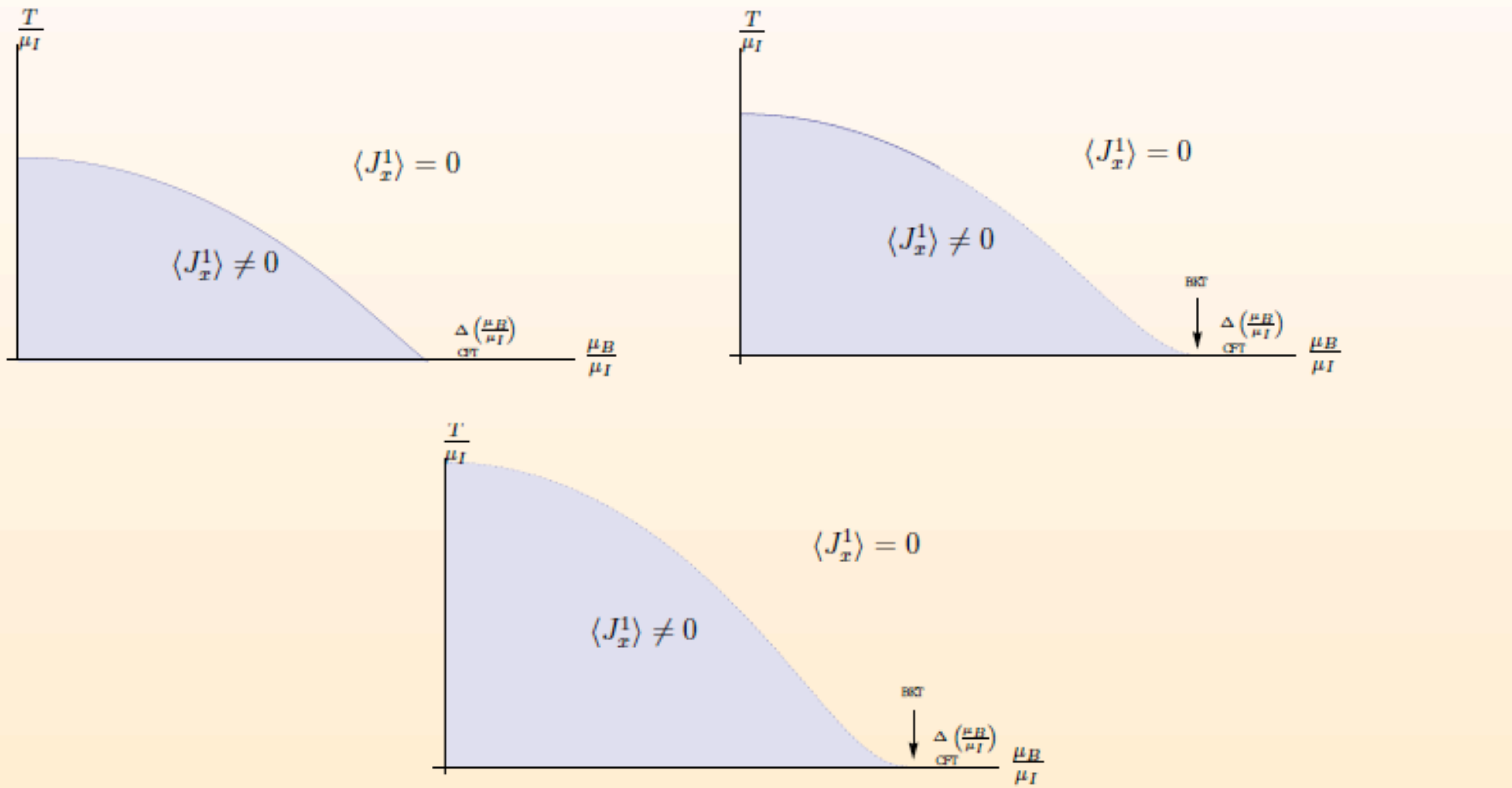
Quantum Phase Transition



Quantum phase transition

Figure by Patrick Kerner

Example with backreaction: $SU(2)$ Einstein-Yang-Mills Model



J.E., Grass, Kerner, Ngo 1103.4145

BKT transition in gauge/gravity duality

Jensen, Karch, Son, Thompson 2010
Evans, Gebauer, Kim, Magou 2010

Order parameter scales as $\exp(-c/\sqrt{T_c - T})$

Gravity side: violation of the BF bound in the IR

IR $AdS_2 \times S^2$ region

Only possible when the two parameters have the same dimension

D3/D7 vs. backreacted model

D3/D7:

Effective IR mass of A_x^1/r vanishes,
independently of μ_B
BF bound violated along flow, but not in IR
Flavor fields directly interact with each other

Einstein-Yang-Mills:

Effective IR mass depends on μ_B/μ_I
BF bound violated in IR
$AdS_2 \times S^2$ region in IR
Flavor fields interact with gluon fields

Conclusion

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect
- Add baryon chemical potential: Imbalanced mixtures
- Quantum critical point arising from AdS_2 in IR